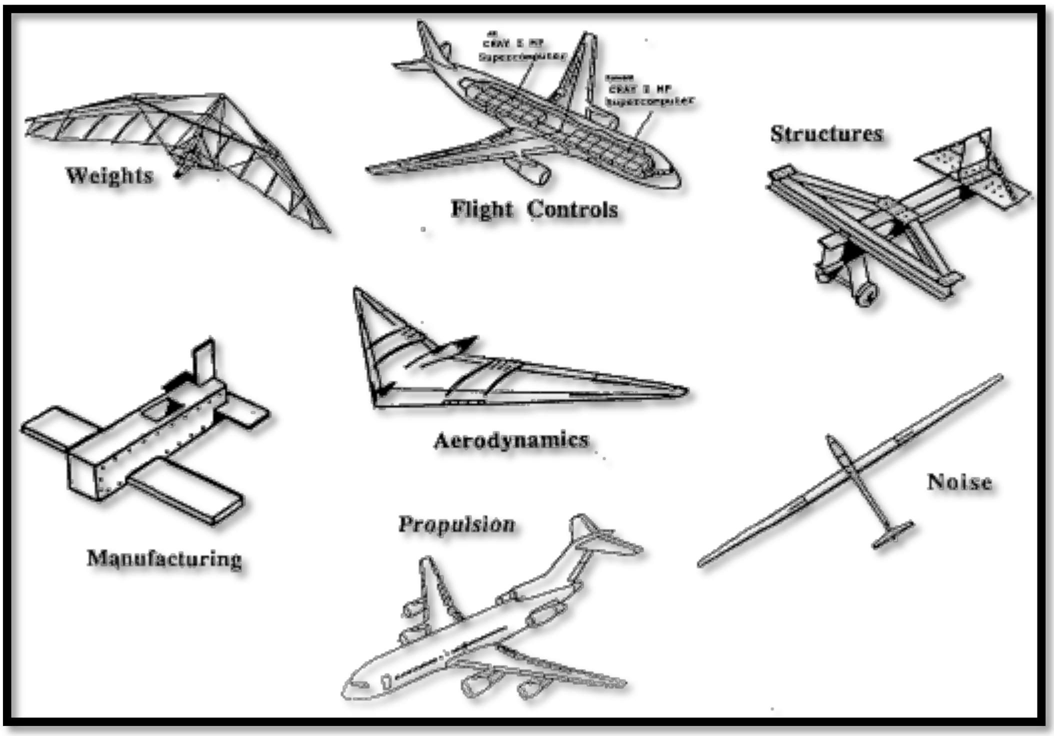
MEng/MSc group task – Aircraft

Group 9



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Abstract

Having chosen the NACA 2408 aerofoil it was investigated whether this wing could be used effectively on an Airbus A380. The equivalent Joukowski aerofoil was analysed as well as the takeoff, cruising and landing conditions of the Airbus with our chosen NACA aerofoil.

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1 Introduction

This project aimed to design an airfoil for an aircraft. From that design, the aerodynamic aspects of the aircraft were to be obtained to give a complete arrangement.

In order to achieve this, initial conditions had to be set and certain assumptions were made utilising the data available for the Airbus A380:

- Flying from Heathrow to Newark
- Fuselage Length (72.72m)
- Fuselage Width (7.14m)
- Wing Span (79.75m)
- Airfoil Chord Length (11m)
- Cruising Speed (262.5m/s)
- Cruising Altitude (40000ft)

2 Airfoil

Identify the Joukowski aerofoil which matches that geometry most closely

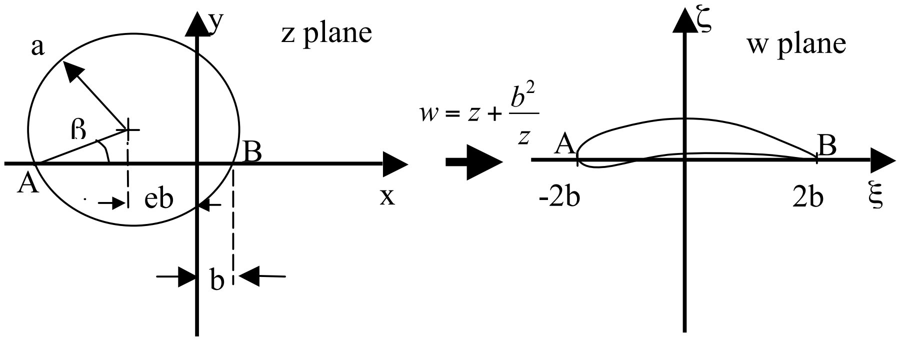


Figure 2-1: Joukowski Transform

By using conformal mapping see figure above, we are able to find the Joukowski aerofoil which matches the best to our NACA profile. Conformal mapping is used in aerodynamics to solve practical aerodynamics application, because by using Joukowski transform the circulation around the cylinder is the same as around the aerofoil.

$$\rho\, V_0\, \Gamma_{Cylinder} = \rho\, V_0\, \Gamma_{Joukowski}$$

To help us to solve this problem we have done a Matlab code capable to compare them see Appendix C-1. We have used the same given such as the thickness which is t=0.08 and the parameter related to the camber is c=0.02.

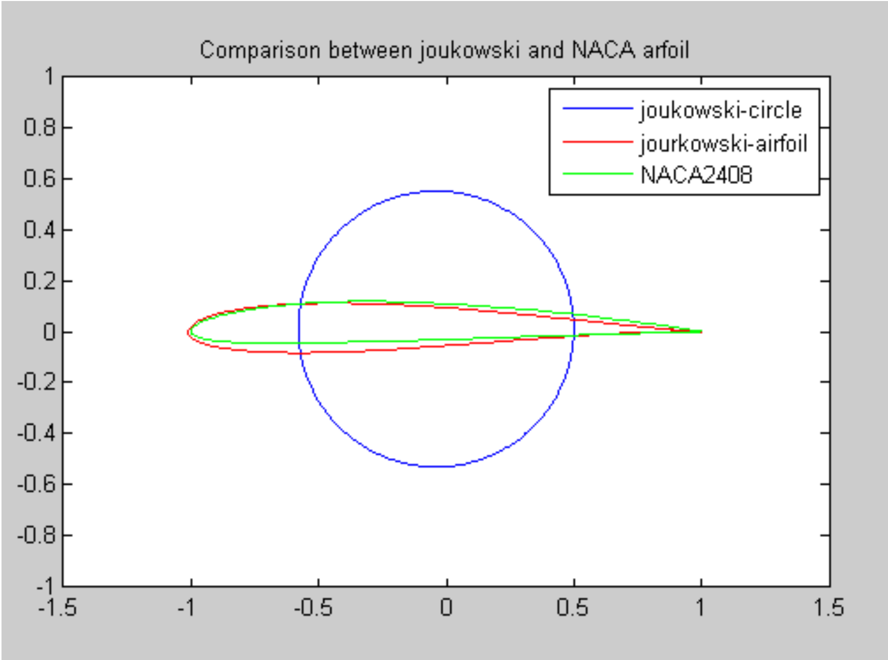


Figure 2.2: Result of the Matlab code

Thanks to this Joukowski transform we are now able to say that the angle $\beta = 1.8°$ and the coordinate of the center of our circle is z=0.02 + 1.08 i

The equation of the offset circle became: $z = 1.08 * e^{i\theta} + 0.02$ with $\theta = [0; 2\pi]$

Joukowski airfoil lifts coefficient calculation:

$$C_L = \frac{8\,\pi\,a}{c}\sin(\alpha+\beta)$$

$$C_L = 0{,}906 \qquad with\ c \approx 4a, \alpha = 5°\ and\ \beta = 1.8°$$

See below the results of calculation of lift coefficient with an angle β = 1.8° and with angles of attack varying between -5° and 20°. We can see that the Lift coefficient varies with the angle of attack but the critical angle of attack is close to 13° with this Joukowski airfoil.

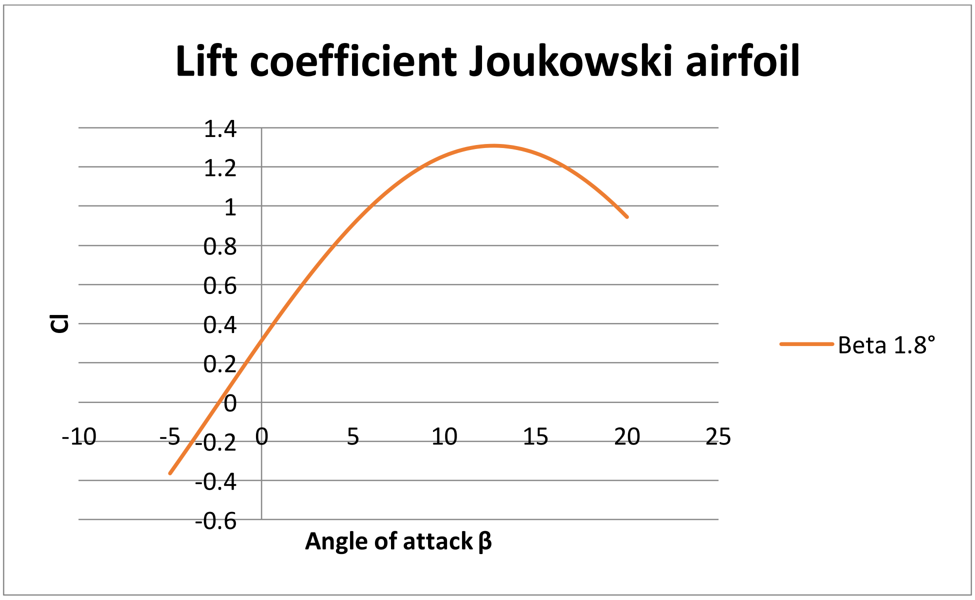


Figure 2.3: Lift coefficient on Joukowski airfoil

Now Lift force calculation:

$F_L = -\rho\, V_0\, \Gamma$
$\mathrm{F_L} = 4\,\pi\, V_0^2\, \rho\, a \sin(\alpha + \beta)$
$F_L = 85584\; N\; per\; unit\; span \qquad with\; \rho = 0.3\,, \alpha = 5°\,, \beta = 1.8°\; and\;\; V_0 = 262.5\; {}^{m}/_{s}$

Which means that the global lift force of our Joukowski airfoil applied to A380 is given by:

$F_L = 79{,}75 * \; 85584$
$F_L = 6{,}74 * 10^6 N$

The Force needed to have the plane at equilibrium is $F = m * g = 5{,}49.\,10^6 N$, its mean that the minimum angle of attack to be at equilibrium is α=3.8°.

2.1 Joukowski

The expected real lift (C_l) and drag coefficient (C_d) were calculated by utilising a software program called 'JavaFoil'. To calculate the lift and drag coefficient the following values were inserted into the program:

- thickness to chord ratio $\frac{t}{c}$ of 8%
- camber to chord ratio $\frac{f}{c}$ of 2%

Once the values above were inserted the program, the program calculated the various values at a number of different angle of attacks (α) and then inserted into a table as shown below in Table 2.2: Lift and drag cofficients for the NACA 2408 Airfoil.

Table 2.1: Lift and drag cofficients for the Joukowski Airfoil

α	Cl	Cd
[°]	[-]	[-]
-5	-0.293	0.04176
-4	-0.126	0.03032
-3	-0.142	0.01104
-2	0.069	0.01042
-1	0.291	0.01364
0	0.512	0.0216
1	0.732	0.03446
2	0.949	0.05159
3	1.135	0.07007
4	1.264	0.08925
5	1.278	0.10697
6	1.4	0.13577
7	1.575	0.16632
8	1.752	0.19917
9	1.924	0.23582
10	2.111	0.27256

2.2 NACA 2408

The expected real lift (C_l) and drag coefficient (C_d) were calculated by utilising a software program called 'JavaFoil'. To calculate the lift and drag coefficient the following values were inserted into the program:

- thickness to chord ratio $\frac{t}{c}$ of 8%
- camber to chord ratio $\frac{f}{c}$ of 2%
- camber location marked at $\frac{xf}{c}$ 40% between the leading edge and the trailing edge

Once the values above were inserted the program, the program calculated the various values at a number of different angle of attacks (α) and then inserted into a table as shown below in Table 2.2.

Table 2.2: Lift and drag cofficients for the NACA 2408 Airfoil

α	C_l	C_d
[°]	[-]	[-]
-5	-0.268	0.03743
-4	-0.133	0.02762
-3	-0.149	0.00894
-2	0.022	0.00829
-1	0.194	0.00819
0	0.366	0.00851
1	0.537	0.00886
2	0.709	0.00939
3	0.88	0.01039
4	1.051	0.01294
5	1.064	0.02811
6	1.161	0.03939
7	1.26	0.05455
8	1.4	0.06432
9	1.544	0.07253
10	1.699	0.07759

3 Finite Wing

3.1 Lift Distribution

Real wings are not of an infinite span. They have a finite wing span, because of this it affects the amount of lift generated considerably compared with an infinite wing. To calculate the lift per unit span over the wing the following equations can be utilised:

$$F_{L,z} = -\rho \Gamma_0 U_0 \int_{\frac{-b}{2}}^{\frac{b}{2}} \sqrt{1 - \left(\frac{z}{b/2}\right)^2}\, dz, \text{ N [1]}$$

Where,

$$U_0 = Freestream\ velosity = 262.5\,\frac{m}{s}$$

$$\rho = density\ of\ air\ at\ 13000m = \ 0.3\,\frac{kg}{m^3}$$

$$\Gamma_0 = circulation = -4\pi U_0 a \sin(\beta + \alpha)$$

$$\alpha = angle\ of\ attack$$

$$\beta = beta\ 1.8$$

$$a = \frac{chord}{4} = \frac{11}{4} = 2.75$$

$$b = span\ of\ the\ wing = 79.75m$$

The elliptical distribution equation above calculates the lift force at a point along the z-axis. Inputting this equation is to a software package such as Excel, can form a graph to give a visual representation of lift distribution, this is shown in the graph below. See Appendix A – Finite Wing (b/2)for the values for each point along the axis.

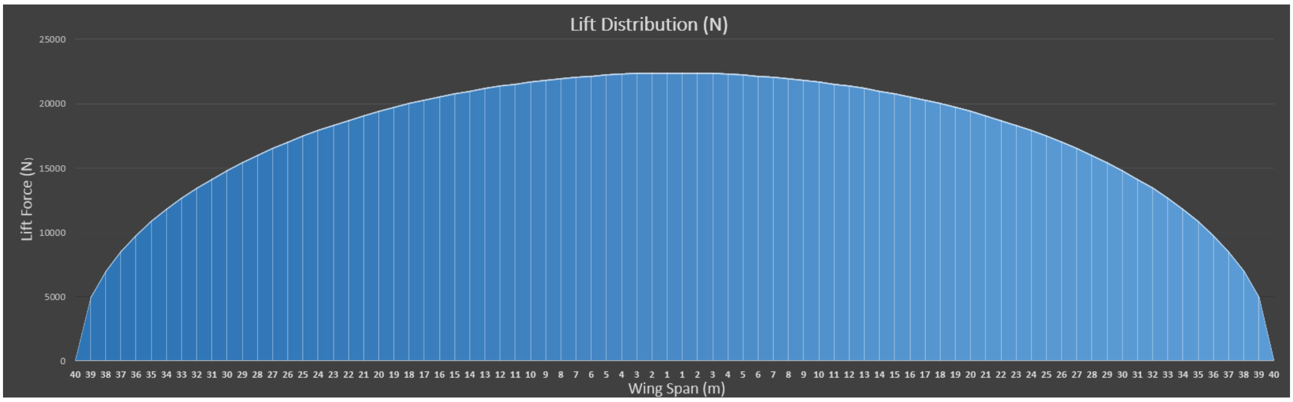


Figure 3.1: Lift distribution

This equation can be simplified for the whole section to $F_L = -\rho \Gamma_0 U_0 \frac{\pi}{4} b$, which equals =5222kN at α =5

Where,

$$\Gamma_0\ at\ 5° = -4\pi U_0 a \sin(\beta + 5) = -\ 1074$$

3.2 Lift Coefficient

To calculate the lift coefficient of the wing at a point along the z-axis the following equation is used [2]:

$$C_{L,w,x} = \frac{F_{L,z}}{{}^{1}\!/_{2}\,\rho U_0^2 c}$$

Where,

C = Chord 11m

The total lift coefficient is calculated by using the following:

$$C_{L,section} = \frac{F_{L,section}}{{}^{1}\!/_{2}\,\rho U_0^2 A} \times \frac{\pi}{4} = 0.1217$$

Where, $F_{L,section}$ = the total force along the wing.

$$A_{area} = 11m \times 80m = 880m^2$$

3.3 Induced Drag

The induce drag is caused when a moving object redirects the air flow coming at it. The change of direction causes the air to produce a downward force. At the wing tips, the pressure flow changes flow where the differential pressure goes from under the wing to over the wing tip. As a result of this vortices are created. The vortices create a rotational flow which results in a downward direct flow, thus creating induced drag. The equation below calculates the force of induced drag along the z-axis:

$$F_{Di} = -\rho \frac{\Gamma_0}{2b} \Gamma_0 \int_{\frac{-b}{2}}^{\frac{b}{2}} \sqrt{1 - \left(\frac{z}{b/2}\right)^2}\, dz, \text{N}$$

This equation is similar to that of the lift distribution equation where if plotted on a graph it will create a parabolic profile, as shown below Figure 3.2: Induced Drag. See appendix Appendix A – Finite Wing (b/2) for the values for each point along the axis.

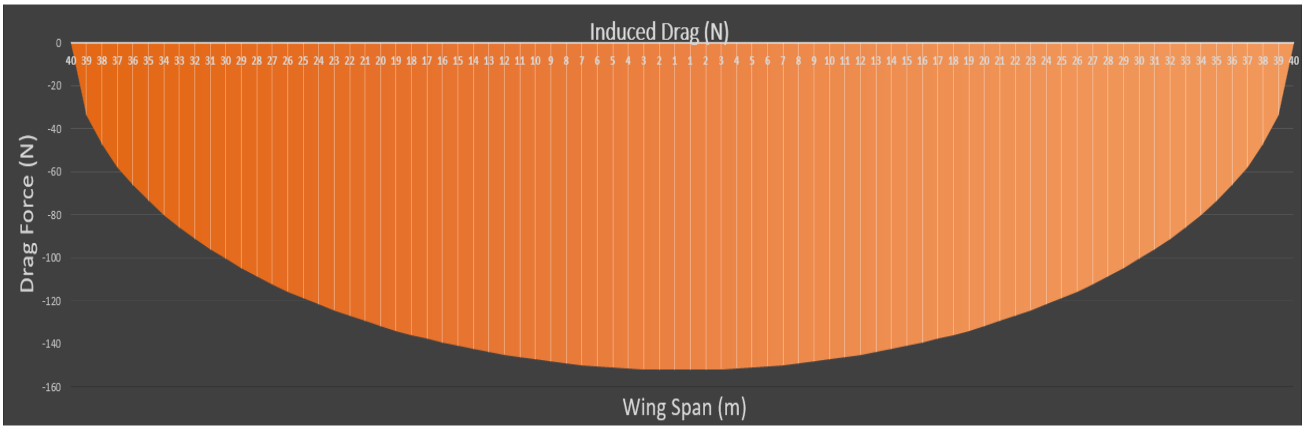


Figure 3.2: Induced Drag

This induced drag equation can be simplified for the whole section to $F_{D,i} = -\rho \Gamma_0{}^2 \frac{\pi}{8}$, which equals -13.6kN at $\alpha = 5$

3.4 Induced Drag Coefficient

The induced drag coefficient for the wing at a point along the z-axis can be calculated with the following [2]:

$$C_{D,i} = \frac{C_{L,w,x}{}^2}{\pi A R e}$$

Below shows the induced drag coefficient distribution at each point along the z-axis:

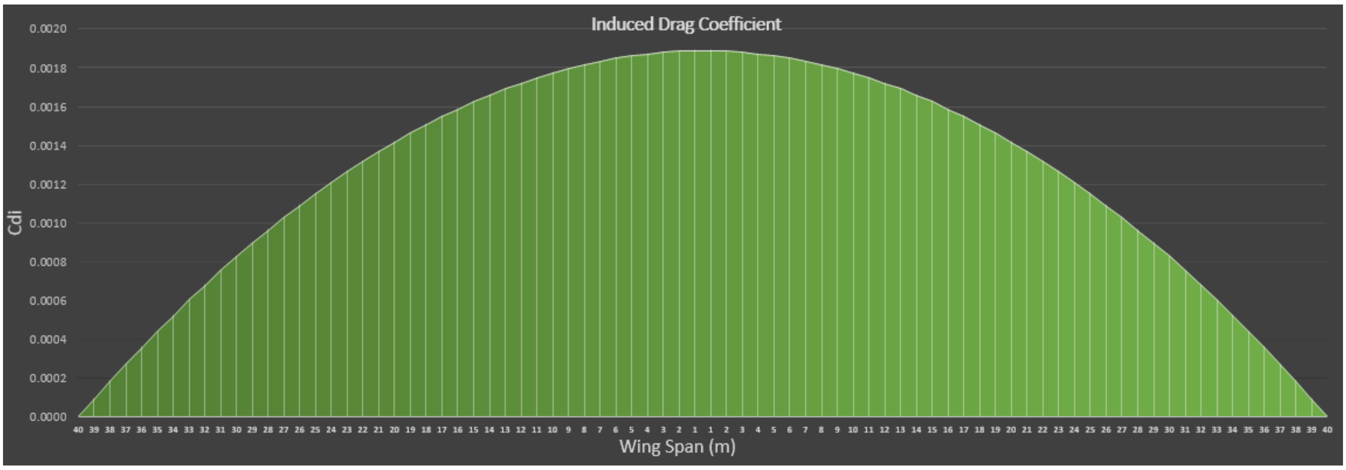


Figure 3.3: Induced drag coefficient

The induced drag for the entire section can be calculated from the following:

$$C_{D,i} = \frac{C_{L,section}{}^2}{\pi A R e} = \frac{0.0148}{20.56} = 7.198 \times 10^{-4}$$

Where,

Aspect ratio, $AR = \frac{span}{chord} = \frac{80m}{11m} = 2.27$

Spanwise efficiency factor, e = 0.9

3.5 Downward wash velocity

The downward wash created from the aerofoil is calculated from the equation below:

$$\mathrm{V_i} = \frac{\Gamma_0}{2\mathrm{b}} = \frac{-284.94}{2 \times 80} = -1.78 \frac{\mathrm{m}}{\mathrm{s}} \text{ [3]}$$

3.6 Skin Friction

The skin friction force is given by the equation:

$$F_s = \frac{1}{2}\rho U_0^2 A c_f$$

The following constants are assumed:

$$U_0 = 262.5 \text{ m/s}^2 \text{ (Cruising speed)}$$

altitude = 40,000 feet

$$\rho = 0.3\ kg/m^3$$

$$\mu = 1.4 \times 10^{-5}\ kg/ms$$

To work out the skin friction force for the wings and fuselage the following needs to be calculate:

- Area
- Reynolds number (used to identify whether it is lamina or turbulent)
- Skin friction coefficient

3.6.1 Fuselage

3.6.1.1 Fuselage length

The fuselage can be viewed as a cylinder, the surface area is calculated as follows:

$$A = \pi DL$$

where L = 72.72 and D = 7.14 [4]

Reynolds number can then be used to work out whether the air is turbulent or laminar. This is given by:

$$R_e = \frac{\rho U_o L}{\mu} = \frac{0.3 * 262.5 * 72.72}{1.4 \times 10^{-5}} = 403 \times 10^6$$

Turbulent flow occurs when R_e is greater than 500000. Therefore the fuselage can be assumed to be in turbulent flow.

The following equation is used to work out the skin friction coefficient for turbulent flow

$$c_f = \frac{0.0711}{R_e^{\frac{1}{5}}} = 1.35 \times 10^{-3}$$

From this it is possible to work out the skin friction force.

$$F_s = \frac{1}{2} \times 0.3 \times 262.5^2 \times 1631.18 \times 1.35 \times 10^{-3} = 22.76\ kN$$

3.6.1.2 Cockpit

The cockpit is front facing and leads the aeroplane into incoming air. As the flow is not parallel to the surface; flat plate conditions cannot be assumed. The circulation around the wings was modelled using the Joukowski aerofoil as an approximate shape of the NACA 2408. For the cockpit drag, a bullet is used where the drag coefficient has been determined experimentally.

$$C_{d,fuselage} = C_{d,bullet} = 0.295\ [5]$$

The friction force is applied against the frontal area of the fuselage which is assumed to be circular:

$$A = \pi r^2 = \pi \left(\frac{7.14}{2}\right)^2 = 40.04 m^2$$

$$F_{s,cockpit} = \frac{1}{2} \times 0.3 \times 262.5^2 \times 40.04 \times 0.295 = 122.09\ kN$$

The total skin friction of the fuselage section is the addition of these two:

$$F_{s,fuselage} = 122.09 + 22.76 = 144.85\ kN$$

3.6.2 Wing

For the wing the area is taken as the area of the top and bottom of the wing. The area is calculated as follows:

$$A = 2 \times span \times chord$$

where span = 79.75 and chord = 11.

Reynolds number can then be used to work out whether the air is turbulent or laminar. This is given by:

$$R_e = \frac{\rho U_o L}{\mu} = \frac{0.3 * 262.5 * 11}{1.4 \times 10^{-5}} = 61 \times 10^6$$

Turbulent flow occurs to occur when R_e is greater than 500000. We can therefor take the wing as being in turbulent flow.

To work out the skin friction coefficient for turbulent flow the following equation is use

$$c_f = \frac{0.0711}{R_e^{\frac{1}{5}}} = 1.97 \times 10^{-3}$$

The skin friction force can now be calculated.

$$F_s = \frac{1}{2} \times 0.3 \times 262.5^2 \times 1631.18 \times 1.97 \times 10^{-3} = 33.2\ kN$$

3.6.3 Rear stabilisers

The Reynolds number under cruising conditions:

$$Re_{horizontal} = \frac{0.3 * 262.5 * 7.5}{1.42 \times 10^{-5}} = 4.159 \times 10^7$$

$$Re_{vertical} = \frac{0.3 * 262.5 * 9.5}{1.4 \times 10^{-5}} = 5.26 \times 10^7$$

Turbulent flow occurs when Re is greater than 500000, in both cases we can assume fully turbulent flow, neglecting the initial laminar and transitional regions. The coefficient of skin friction can then be calculated and hence the skin friction:

$$c_{f,horizontal} = \frac{0.0711}{Re_{horizontal}^{\frac{1}{5}}} = 0.002126$$

$$c_{f,vertical} = \frac{0.0711}{Re_{vertical}^{\frac{1}{5}}} = 0.0020274$$

$$F_{s,horizontal} = \frac{1}{2} \times \rho \times U_0^{\ 2} \times A \times C_f$$

Each surface area of the wing is required:

$$A_{horizontal} = 2 \times 30.37 \times 7.5 = 455.55 m^2$$

$$A_{vertical} = 2 \times 14.59 \times 9.5 = 277.21 m^2$$

The friction for each stabiliser:

$$F_{s,horizontal} = \frac{1}{2} \times 0.3 \times 262.5^2 \times 455.55 \times 0.002126 = 10.008\ kN$$

$$F_{s,vertical} = \frac{1}{2} \times 0.3 \times 262.5^2 \times 277.21 \times 0.0020274 = 5.809\ kN$$

These stabilisers are symmetric aerofoils, the coefficient of lift is zero for zero angle of attack therefore the induced drag is also zero. The total skin friction for the tail section is simply:

$$F_{s,tail} = F_{s,horizontal} + F_{s,vertical} = 10.008 + 5.809 = 15.817 kN$$

We can now add these together to get the total skin friction:

$$F_{s,total} = F_{s,fuselage} + F_{s,wing} + F_{s,tail} = 144.85 + 33.2 + 15.817 = 193.87kN$$

The force required in cruising conditions for each engine is therefore:

$$F_{engine} = \frac{F_{s,total}}{No.\,of\ engines} = \frac{193.87}{4} = 48.468kN$$

3.7 Total Drag Coefficient

3.7.1 Wing

The total drag coefficient for the wings is calculated by adding up the following:

$$Total\ drag\ coefficient = c_f + c_p + c_d$$

where

$c_f = skin\ friction\ drag\ coefficient\ (viscous\ drag)$

$c_p = pressure\ drag\ coefficient\ (form\ drag)$

$c_d = induced\ drag\ coefficient$

The profile drag is given by $c_f + c_p$. Since it is very hard to calculate the pressure drag coefficient, we will get the profile drag from 'Javafoil'. This is given by 0.01039.

The drag coefficient is given by the equation:

$$c_d = \frac{c_L^2}{\pi \times AR \times e}$$

where

$$AR(Aspect\ Ratio) = \frac{span}{chord} = \frac{79.75}{11} = 7.25$$

this is assuming that the wing has a constant chord length.

e is the span efficiency factor. It is possible to assume Oswald efficiency factor = 0.9 [6].

From the table of previously calculated lift coefficient for different wing span, assuming an angle of attack of 3° , the total drag coefficient of the wing can be calculated.

From the appendix D.1 the total drag coefficient of the wing can be seen to be roughly 0.02383.

3.7.2 Stabilisers

In addition to the drag of the wings the fuselage and stabilisers add friction drag:

$$c_d = c_f + c_p$$

where

$c_f = skin\ friction\ drag\ coefficient\ (viscous\ drag)$

$c_p = pressure\ drag\ coefficient\ (form\ drag)$

In this case the drag coefficient can be calculated by JavaFoil which required the Reynolds number of the flow around them.

$$Re_{h\ rear} = 42187500$$

$$Re_{v\ rear} = 53437500$$

$$c_{d,h\ rear} = 0.0053$$

$$c_{d,v\ rear} = 0.0052$$

3.8 Total Drag

The total drag is the addition of all the components of drag. As the fuselage is not a aerofoil the drag coefficient is not given by JavaFoil and the skin friction equations above are assumed.

$$F_{d,component} = \rho C_d {U_0}^2 A_{component}$$

$$F_{d,fuselage} = F_{s,length} + F_{d,front} = 22.7 + 224.2 = 246.9kN$$

$$F_{d,v\ rear} = 14.9kN$$

$$F_{d,h\ rear} = 25.0kN$$

$$F_{d,wing} = 94.7kN$$

$$F_{d,induced} = 54.4kN$$

$$F_{total} = 246.9 + 14.9 + 25.0 + 94.7 + 54.4 = 455.8kN$$

4 Boundary Layers

4.1 Flat Plate Analysis

LAMINAR-TURBULENT REGION

Modelling as a flat plate, the laminar-turbulent region and boundary layers of the airfoil section were found.

Onset of the transition region was found from the Reynolds Number equation. Given that the turbulent region begins when Re=360000:

$$Re_T = \frac{\rho U_0 x_T}{\mu}$$

$$x_T = \frac{360000\mu}{\rho U_0}$$

$$x_T = \frac{(360000)(0.0000142)}{(0.3)(262.5)}$$

$$x_T = 0.0649m$$

The fully turbulent region occurs when Re=500000:

$$Re_C = \frac{\rho U_0 x_C}{\mu}$$

$$x_C = \frac{500000\mu}{\rho U_0}$$

$$x_C = \frac{(500000)(0.0000142)}{(0.3)(262.5)}$$

$$x_C = 0.0902m$$

BOUNDARY LAYER

The Boundary Layers of each region were found assuming a Blasius Profile could be applied. At the end of the Laminar flow:

$$\delta_L = \frac{5}{\sqrt{Re}} x$$

$$\delta_L = \frac{5}{\sqrt{360000}}(0.0649)$$

$$\delta_L = 0.000541m$$

Turbulent Flow conditions begin at 0.0902m along the wing length, and the associated boundary layer will grow as the length increases to the maximum chord length of 11m. At the onset of Turbulent Flow:

$$\delta_{T,S} = \frac{0.366}{Re^{0.2}} x$$

$$\delta_{T,S} = \frac{0.366}{(500000)^{0.2}}(0.0902)$$

$$\delta_{T,S} = 0.00239m$$

For chord length 11m and Re=61000000:

$$Re = \frac{(0.3)(262.5)(11)}{0.0000142} = 61000000$$

$$\delta_{T,E} = \frac{0.366}{Re^{0.2}} x$$

$$\delta_{T,E} = \frac{0.366}{(61000000)^{0.2}}(11)$$

$$\delta_{T,E} = 0.1116m$$

The plane itself can be analysed as a flat plate, giving the boundary layer at the tail section:

$$Re = \frac{(0.3)(262.5)(72.72)}{0.0000142} = 403300000$$

$$\delta_{Tail} = \frac{0.366}{(403300000)^{0.2}}(72.72)$$

$$\delta_{Tail} = 0.506m$$

Figure 4.1 shows the height of the boundary layer for the airfoil.

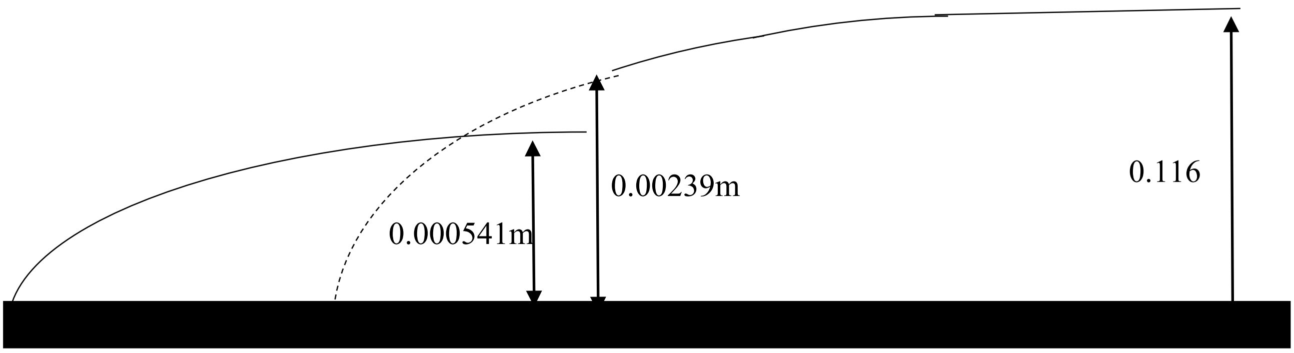


Figure 4.1: Flat plate of an aerofoil

5 Take-off

To model the take-off of an aeroplane a kinematic model is considered as shown in Figure 5-1. In order to calculate the equivalent lift requirement of the wing, the moment of each lift force from the force of gravity must be determined. The tail lift and the main wing lift must be balanced against the centre of gravity (CoG) to keep it flying straight. The equilibrium is essential on any aircraft because it ensures the aircraft does not rotate and dive. On passenger aircraft the CoG changes position regularly, generally manufacturers will provide an acceptable range of loading configurations to allow for adaption of the mass centre and the counteracting mechanisms used by the aeroplane such as the rear wing flap deflection.

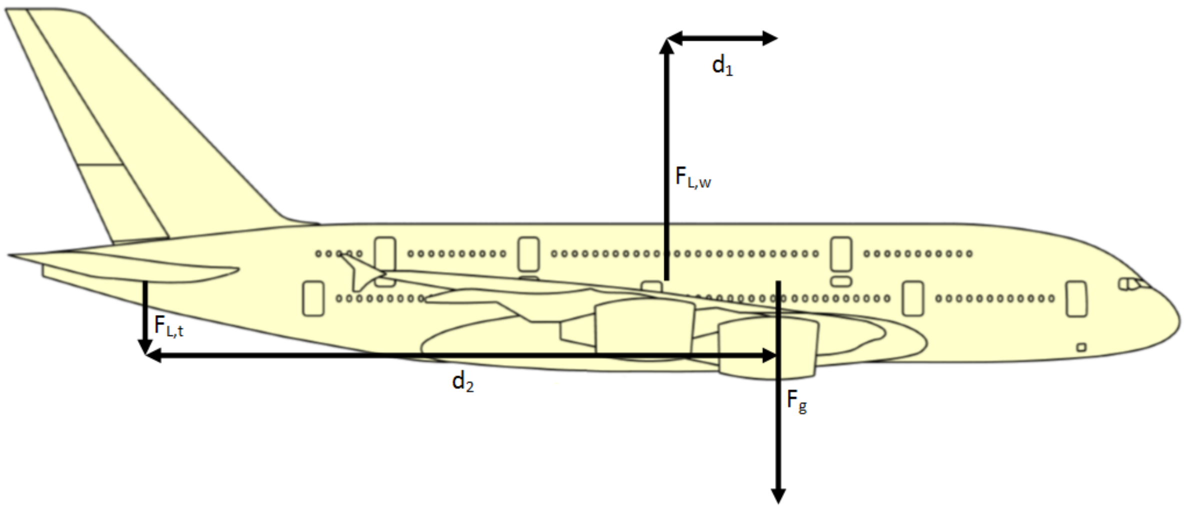


Figure 5.1: Vertical forces and their position across the length of the aircraft. [7]

The main forces and distances are given below with the following assumptions:

$F_g = 22.23m$, parallel to the inboard engines, measured from the nose. [7]

$d_1 = 7.71m$, parallel to the outboard engines, measured from the CoG datum. [7]

$d_2 = 40.835m$, assuming that lift force is uniform from the centre of the wing. [7]

To find the lift requirements of each wing we assume the equilibrium of forces and use Newton's second law calculate the force:

$$0\ F_g = 7.71\ F_{L,w} + 40.835\ (-F_{L,t})$$

$$F_{L,t} = \frac{7.71}{40.835} F_{L,w}$$

$$F_g = F_{L,w} - F_{L,t} = \left(1 - \frac{7.71}{40.835}\right) F_{L,w} = 0.8112\ F_{L,w}$$

$$F_{L,w} = 1.2328\ F_g$$

$$F_g = mg = 577000 \times 9.807 = 5659kN$$

$$F_{L,w} = 6976kN$$

$$F_{L,t} = 1317kN$$

The lift required from the wing is used to find the take-off speed of the aircraft. The lift equation can be rearranged for the speed that this lift can be achieved:

$$F_l = \frac{1}{2}\rho {v_{to}}^2\ c_{l,wing}\ A_{wing}$$

$$A\ =\ length\ x\ chord = 79.75 \times 11\ =\ 877.25\ m$$

$C_l = 1.56$ at 20 degree flap deflection calculated with JavaFoil [8].

$F_l = 6976\ kN$

$\rho = 1.225\ kg/m^3$ the standard density of air at ground level.

$$v_{take-off} = \sqrt{\frac{2F_l}{\rho A c_l}} = \sqrt{\frac{2\times6976000}{1.225\times877.25\times1.56}} = 91.23\ ms^{-1}$$

The take-off speed is the minimum speed at which this aeroplane can provide enough lift to leave the ground. A range is usually given for speeds that are safe to take-off at and the minimum is likely to be higher than this speed to allow for different conditions of take-off. To calculate the thrust force required from the engines to achieve sufficient acceleration, within the limits of the runway, we utilise the fundamental equations of motion. The addition of either a distance or time restriction is required; for most applications a distance is easier to limit as the runway is a set distance. In this case a distance range of 2000 to 3000m coincides with Heathrow and Newark airports which both have runways exceeding 3km [9] [10].

$$t = \frac{v-u}{a}$$

$$s = \frac{1}{2}at^2 = \frac{a}{2a^2}(v-u)^2 = \frac{v^2}{2a}$$

$$F = ma = m\left(\frac{v^2}{2s}\right)$$

The engine requirements for: take-off speed achieved at three kilometres and safely within the runway length at two kilometres. The ratio of the forces is the inverse of the ratio of the distances as the force of acceleration is inversely proportional to distance.

For: S = 3000m

$$F_{min} = 577000\left(\frac{91.23^2}{2\times3000}\right) = 800.366kN$$

$$F_{max} = \frac{3}{2}F_{min} = \frac{3\times800.366}{2} = 1.201kN$$

Under these assumptions four engines of 300.2kN will provide enough power for take-off within two kilometres. The Airbus a380 is available with four of Engine Alliance's GP7200 engines rated at 311kN each and would provide sufficient thrust force alone for this aircraft also. However, when the free-body diagram Fig. 4.2.2 is analysed, there are some issues with this method.

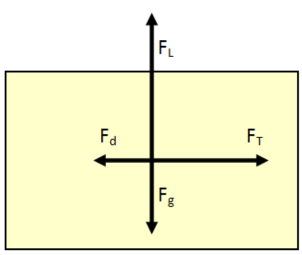


Figure 5.2: Free-body diagram showing the forces acting upon the aeroplane.

Forces act in both x and y axes in positive and negative directions: lift force is reduced by the weight and the thrust force is reduced by the drag. Allocating F_T as the positive direction of x, the real accelerating force is given as:

$$F_a = F_T - F_d$$

Where:

F_a is the acceleration force,

F_t is the combined thrust of all the engines

And, F_d is the total drag force

The total drag force of the aircraft is very difficult to predict and usually determined experimentally within a wind tunnel using a scale model and non-dimensional fluid analysis. This method is costly and accurate to a certain point. There are other methods used to predict a likely drag force numerically using computer programs that can determine the parasitic drag, the induced drag can be determined mathematically and added to this. The application 'JavaFoil' is a Java applet that can be used to determine the flow characteristics of aerofoils and is available for free [8]. Parasitic drag initially is zero but increases with speed of the aircraft because the Reynold's number of the surrounding flow increases. Using a combination of MatLab and JavaFoil the friction of the aircraft has been modelled as the aircraft accelerates to take-off speed (E). Using the friction for each time the acceleration achieved using certain engine thrust is determined. Thus the time taken to reach the take-off speed and the distance travelled over that time period is also calculated. Using the following assumptions the acceleration, speed, distance and force are shown for three engine configurations: the four of Engine Alliance's GP 7200 [11] engines used in the Airbus-A380; Engine Alliance's improved GP7277 [11] engines also used for the Airbus-A380 and four General Electric GE90-115B [12] engines used in the Boeing-777.

$F_{take-off} = 6976kN$

Thrust force:

$F_{t,GP7200} = 4 \times 311 = 1244\ kN$

$F_{t,GP7277} = 4 \times 374 = 1496\ kN$

$F_{t,GE90} = 4 \times 420 = 1680\ kN$

Dimensions:

$L_{fuselage} = 72.72m$

$L_{main\ wing} = 79.75m$

$L_{rear\ horizontal} = 30.37m$

$L_{rear\ vertical} = 14.59m$

$D_{fuselage} = 7.14m$

$c_{main} = 11m$

$c_{rear,h} = 7.5m$

$c_{rear,v} = 9.5m$

The chosen aerofoil is from the NACA series, the 2408. The lift characteristics are shown below with a flap deflection of twenty degrees which yielded high lift whilst maintaining a low drag coefficient:

$C_l = 0.248$ [8]

$C_{l,flap} = 1.56$ [8]

The program starts by calculating the acceleration of the aircraft at time 0 and yields the speed after the first time interval. The speed is used to calculate the Reynolds number of the surrounding flow.

$$Re = \frac{\rho\, U_0\, L}{\mu}$$

Where:

$U_0 = relative\ speed\ of\ air\ flow$

$\rho = 1.225\ kg\ m^{-3}$

$\mu = 1.73\ kg\ m^{-1}s^{-1}$

$L = chord\ length$

The Reynold's number of each wing is computed for each speed and a switch is used to find the drag coefficient, previously found by Javafoil, for each value of Re. To find the drag of the fuselage a different technique is used. The front face is compared to a bullet where the drag coefficient is determined experimentally as 0.295 [5]. The length of the fuselage is assumed a flat plate with length πD, the circumference of the fuselage and the coefficient of drag is given by:

$$C_{d,fuselage} = \frac{0.0711}{Re^{0.2}}$$

As the wing provides lift tip vortices form an induced drag as the aircraft accelerates which increases the coefficient of wing drag with the following dependencies:

$$C_{d,i} = \frac{{C_l}^2}{\pi \times AR \times e} = \frac{0.248^2}{\pi \times \left(\frac{79.75}{11}\right) \times 0.9} = 0.003$$

Where AR is the aspect ratio of the wings and the Ostwald efficiency factor for the Airbus A380:

$e = 0.9$

Using the drag coefficients the drag force is determined at each speed remembering that the area used for the wings is doubled for both top and bottom sections:

$$F_d = \frac{1}{2}\rho {U_0}^2 A C_d$$

The total drag force is the addition of each component drag however the landing gear and runway friction have not been included in the program. Instead a twenty percent increase in total drag is assumed for the landing gear form and friction drag including the rolling friction on the runway. The force equation derived from the free body diagram is hence satisfied and the new acceleration calculated:

$$a = \frac{F_T - F_d}{m}$$

For each second the current lift force that would be generated if the flaps were at 20 degrees is calculated using the lift equation above, however it is reduced by a factor of 0.68 due to the parabolic distribution of force over the finite wing where the tips generate zero lift and the fuselage does not add any lift. Once the lift generated by the wings surpasses 6976kN the program ends with the following results: Fig. 4.2.3, the speed-time graph for each engine; Fig. 4.2.4, the acceleration-time graph for each engine; Fig. 4.2.5, the power-time graph of the engines found by the engine force multiplied by velocity; Fig. 4.2.6, the total energy lost over time found by the drag force times distance travelled.

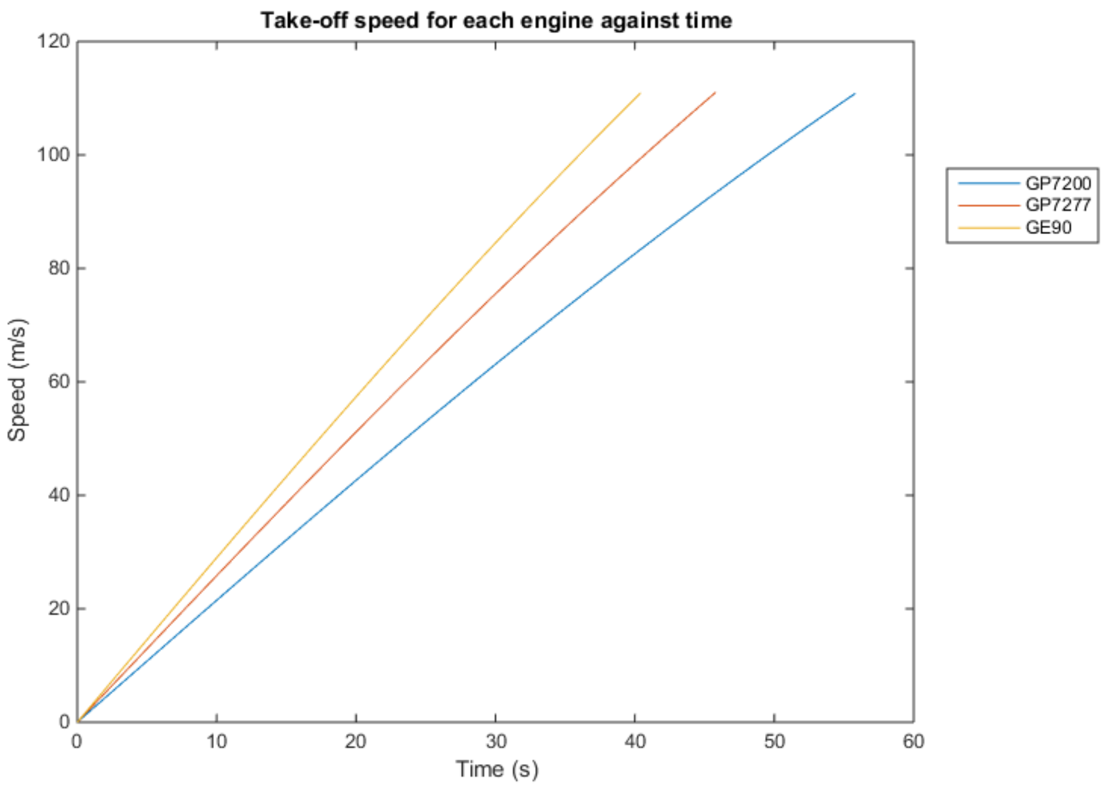


Figure 5.3: Shows the time each engine takes to reach take-off speed

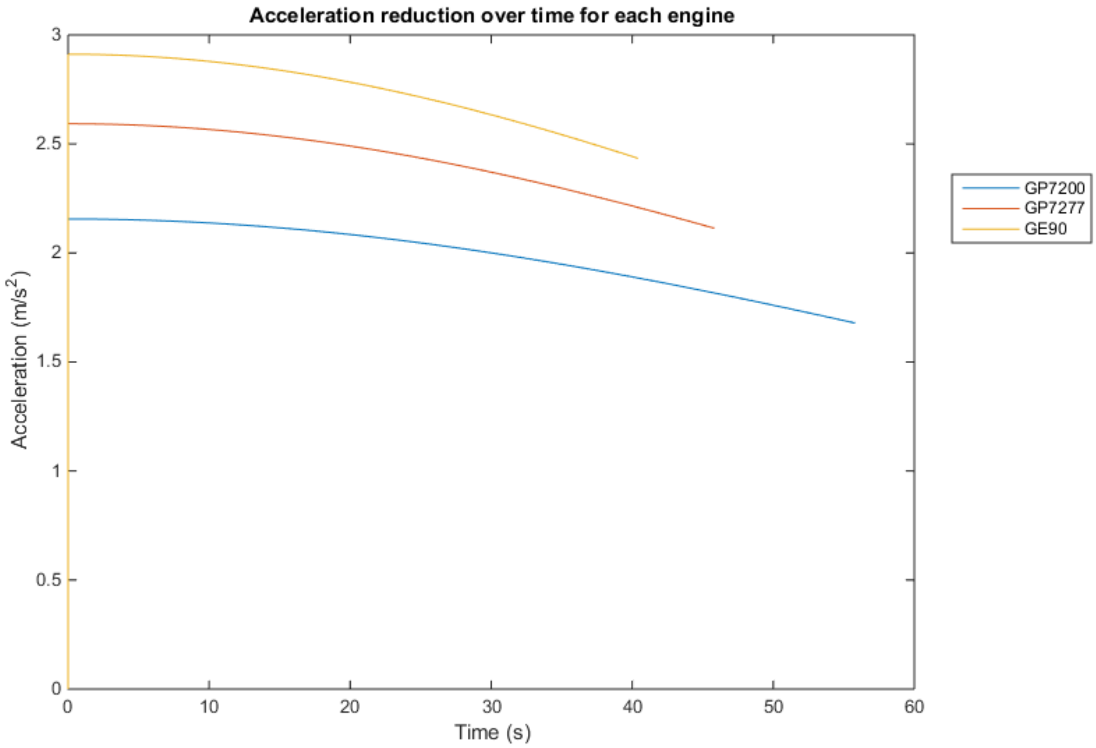


Figure 5.4: Indicates the reduction in acceleration due to increased friction at higher speeds.

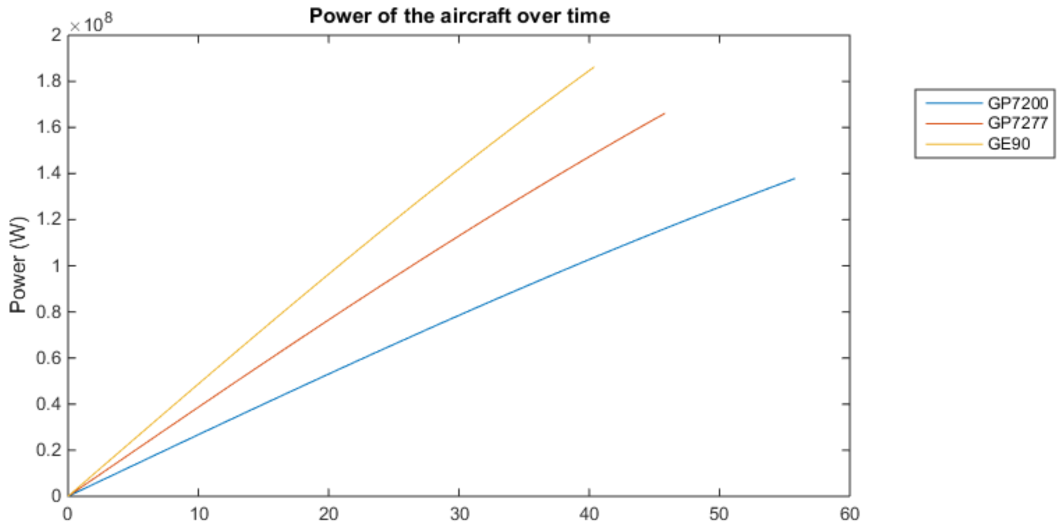


Figure 5.5: Shows the aircraft power with each engine configuration.

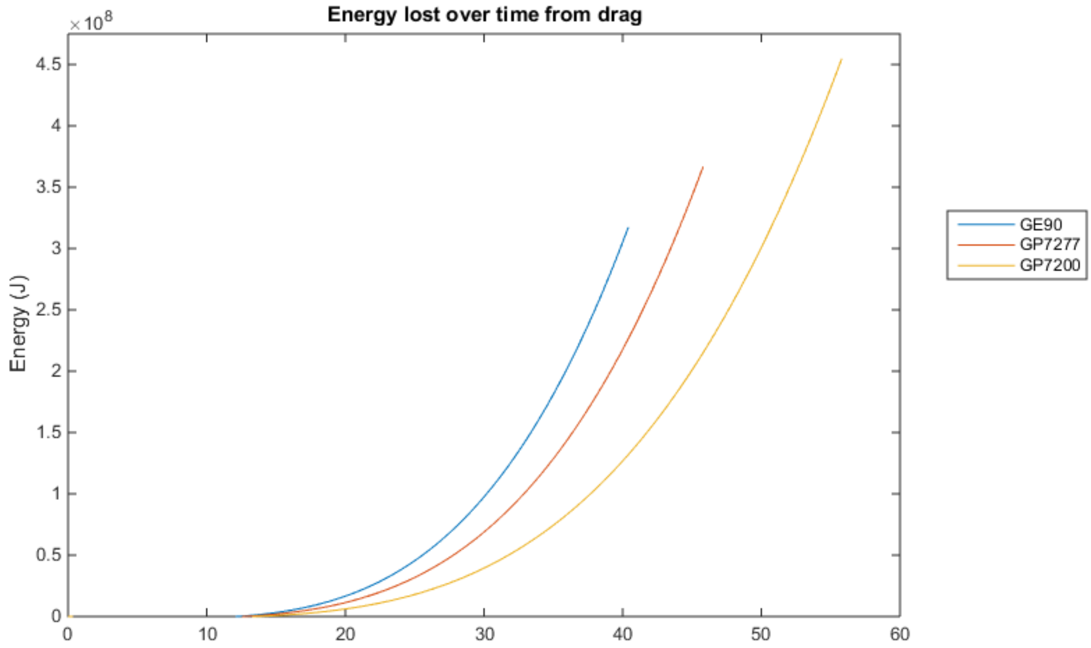


Figure 5.6: Shows the total potential energy of the engines – kinetic energy of the aircraft

The results from the program indicate that the high powered GE90 engines used for the B-777 result in less power loss than with the GP7200 supplied with the A380. The program has its limitations; it is well documented that Engine Alliance's engines have brought a superior level of fuel consumption and are highly efficient engines. Therefore the recommend engines for this craft are Engine Alliance's improved GP7277 engines with a rated power of 374kN with two on each wing, the same configuration used in the A380F.

6 Landing

6.1 Approach

To land successfully, an aircraft has to descend from its cruising altitude. Upon descent, the angle of attack becomes negative, inducing negative lift. Javafoil was used to simulate the drag and lift coefficients produced by an aerofoil at angles of attack ranging from 0˚ to -10˚.

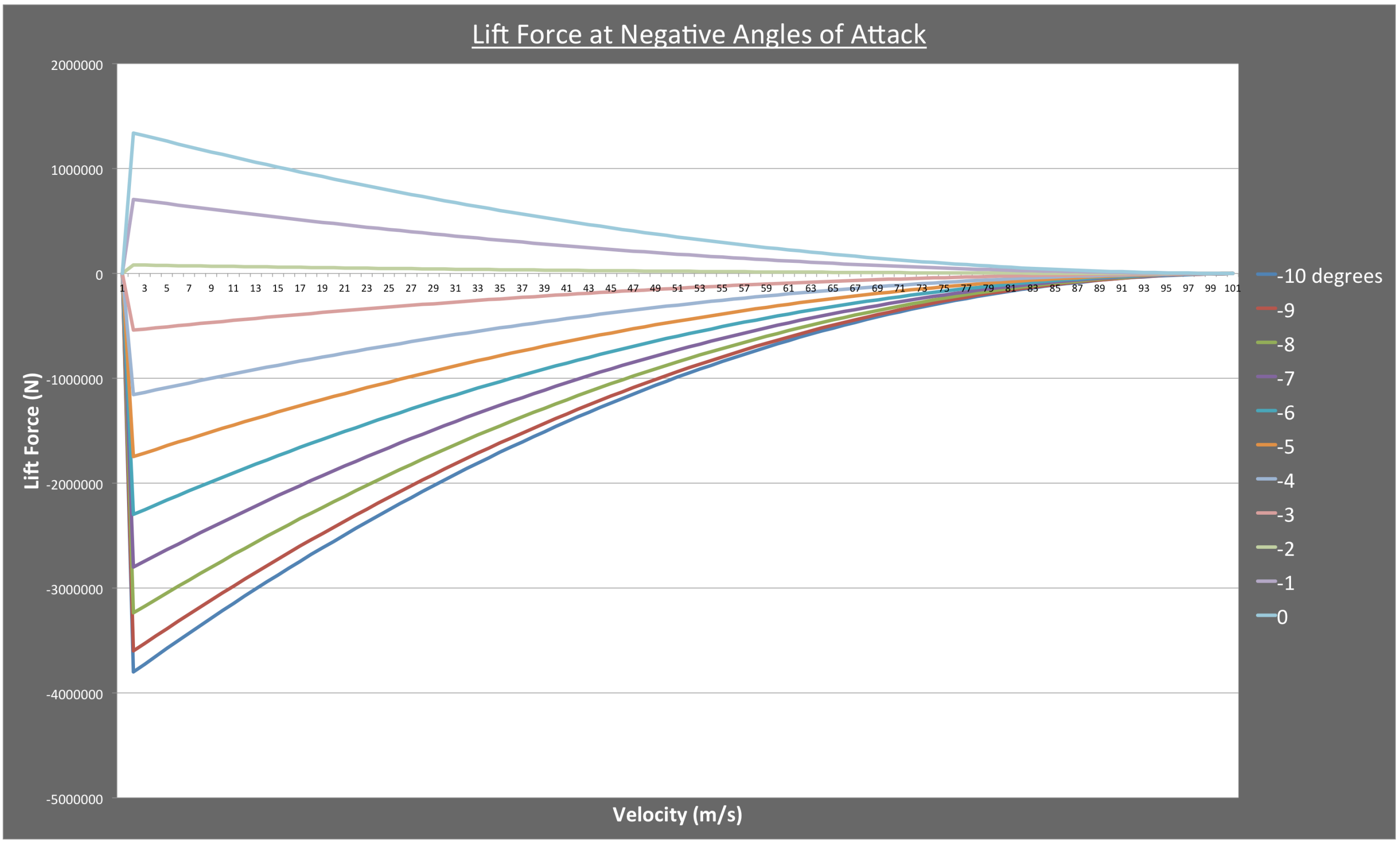


Figure 6-1: Lift Force Against Velocity for Angle Range

This graph shows the lift force produced for ten negative angles of attack at a velocity range of 0 – 100 m/s. At angles of -2˚ upwards, positive coefficients are induced because the airfoil's cambered design encourages positive lift forces. For a symmetric wing, any negative angle of attack will induce negative lift.

After reducing altitude with a negative angle of attack, most cases require a positive angle just before landing. This is to slow the aircraft's descent and allow the first point of touchdown to be the strongest section of the landing gear towards the back.

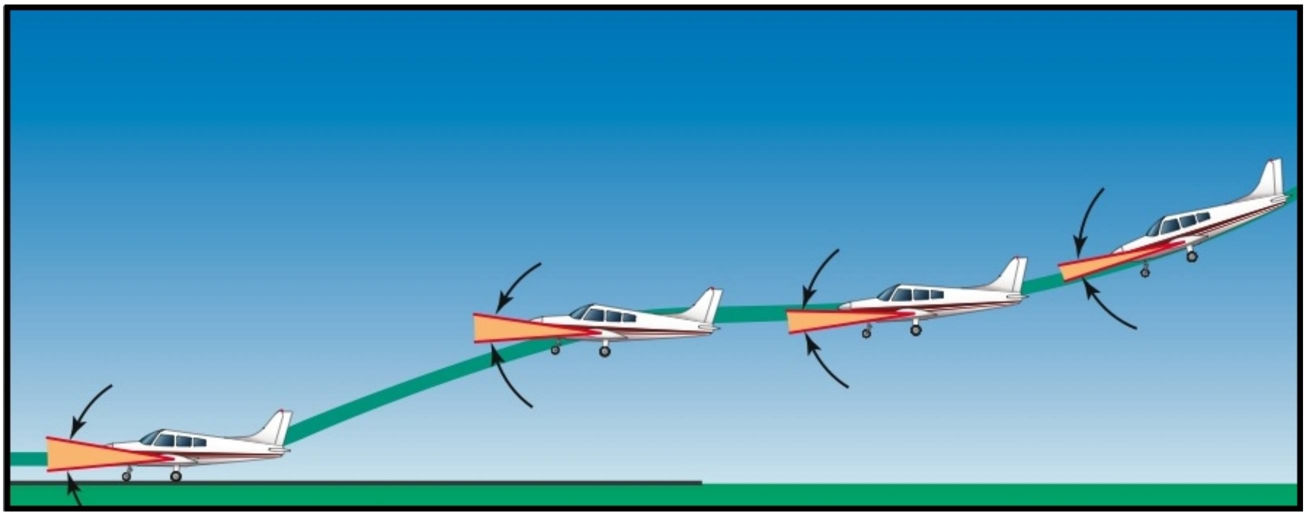


Figure 6-2: Typical Approach of an Aircraft [16]

6.2 Deceleration

Taking the mass of the aircraft as the maximum an A380 can land with, and using the highest suitable velocity and deceleration upon touchdown, the shortest possible stopping distance has been calculated. All calculations have been simplified by assuming constant acceleration and forces.

Starting with the equation of motion for velocity:

$$v = \frac{dr}{dt} = at + u$$

$$dr = v.dt = (at + u).dt$$

Integrating both sides to find distance travelled:

$$\int v.dt = \int at.dt + \int u.dt$$

$$s = a\frac{t^2}{2} + ut$$

Re-arranging for time:

$$t^2 = \frac{2s - 2ut}{a}$$

Squaring equation for velocity to remove the time component:

$$v = at + u$$

$$v^2 = (at + u)(at + u) = a^2t^2 + u^2 + 2t(a.u)$$

$$v^2 = a^2\left(\frac{2s - 2ut}{a}\right) + u^2 + 2t(a.u)$$

$$v^2 = 2as - 2aut + u^2 + 2aut$$

$$v^2 = u^2 + 2as$$

$$\therefore\ s = \frac{v^2 - u^2}{2a}$$

The maximum velocity, weight and deceleration acceptable for a landing A380 have been stated in the Airbus manual. [7] These maximums are related to the landing gear strength and must not be exceeded.

Maximum values for landing:

Approach Velocity	Total Mass	Deceleration
138 knots (71 m/s)	395,000 kg	3.048 m/s^2

$a_{max} = 3.048\ m/s^2$

$v = 71\ m/s$

$u = 0\ m/s$

$s_{max} = 3353\ m$

Using maximum deceleration, the shortest possible stopping distance:

$$s_{min} = \frac{v^2 - u^2}{2a_{max}}$$

$$s_{min} = \frac{71^2 - 0}{2 \times 3.048} = 826.94\ m$$

The associated stopping time for maximum deceleration:

$$t = \sqrt{\frac{2s - 2ut}{a}}$$

$$t_{min} = \sqrt{\frac{2 \times 826.94 - 0}{3.048}} = 23.294\ s$$

The minimum required deceleration has been calculated using the length of Newark runway, 3353m. This assumes that the aircraft can use every metre of the runway and is to be taken as an absolute minimum.

$$s = \frac{v^2 - u^2}{2a}$$

$$\therefore\ a = \frac{v^2 - u^2}{2s}$$

$$a = \frac{71^2 - 0}{2 \times 3353} = 0.7517\ m/s^2\ deceleration.$$

As the minimum deceleration will have the longest stopping time:

$$t_{max} = \sqrt{\frac{2 \times 3353 - 0}{0.7517}} = 94.451\ s$$

6.3 Drag Forces

Maximum and minimum required forces for each deceleration:

$$F = ma$$

$$F_{max,stop} = 395000 \times 3.048 = 1.2\ MN\ (t = 23.29\ s)$$

$$F_{min,stop} = 395000 \times 0.7517 = 296.9\ kN\ (t = 94.45\ s)$$

Javafoil's 'flap deflection' feature simulates the effect of wing flap position on drag force. A deflection of 20° has been applied to the aerofoil to act as a brake after touchdown. A range of values have been calculated using excel but in this example the velocity is 60 m/s:

$$F_d = \frac{1}{2} C_d A \rho U_0^2$$

$$F_D = 0.5 \times 0.01384 \times 879.45 \times 1.229 \times 60^2 = 49.942\ kN$$

Induced drag exists due to lift forces forming vortices around the edges of the aerofoil. For this example AR is 7.25 and e is 0.9. The lift coefficient of the aerofoil with flap deflection 20˚ is 1.968.

$$C_{Di} = \frac{\left(C_{l,wing}\right)^2}{\pi ARe}$$

$$C_{Di} = \frac{(1.968)^2}{\pi \times 7.25 \times 0.9} = 0.18894$$

$$\therefore\ F_{Di} = 0.5 \times 0.18894 \times 879.45 \times 1.229 \times 60^2 = 367.58\ kN$$

The fuselage drag has been calculated using flat plate analysis of the outside area of a cylinder, diameter 7.14m. This is the diameter of the A380 fuselage. The frontal area has been modeled using the drag coefficient of a bullet shape and the same diameter as the fuselage.

$$F_f = 5139.6526 + 26129.597 = 31.26925\ kN$$

For this aircraft, the landing gear causes a 20% increase in total drag. Therefore at 60 m/s, shortly after landing, the drag force on the whole aircraft is:

$$F_{D,tot} = 1.2 \times \left(F_f + F_t + F_{tw} + F_{w,20} + F_i\right)$$

$$F_{D,tot} = 1.2 \times (31.27 + 0.656 + 1.729 + 49.942 + 367.583) = 541.42\ kN$$

6.4 Braking Force

To find the forward momentum of the aircraft, the force generated during the full deceleration period has been calculated.

$$Ft = m_1 v_1 - m_2 v_2$$

$$F = \frac{m_1 v_1 - 0}{t} = \frac{395000 \times 71}{23.29} = 1.2\ MN$$

The brakes used on an A380, Honeywell Carbenix, can produce so much braking force that using reverse engine thrust is not essential to slow the aircraft. [13] At 60 m/s during maximum deceleration, the required braking force for all wheels:

$$F_{brake} = F_{max} - F_{D,tot}$$

$$F_{brake} = 1.2\ MN - 541.42\ kN = 658.59\ kN$$

The A380 uses sixteen braked wheels. They are applied at different times while stopping to maintain a smooth deceleration, however for this analysis it is assumed they are all being applied at once.

$$F_{brake,in} = \frac{F_{brake}}{16} = 41.16\ kN$$

7 Power Fuel

Assuming that for the A380 take-off speed is 280km/h = 77.8 m/s and also during his take-off we will consider a uniform acceleration to simplify calculation.

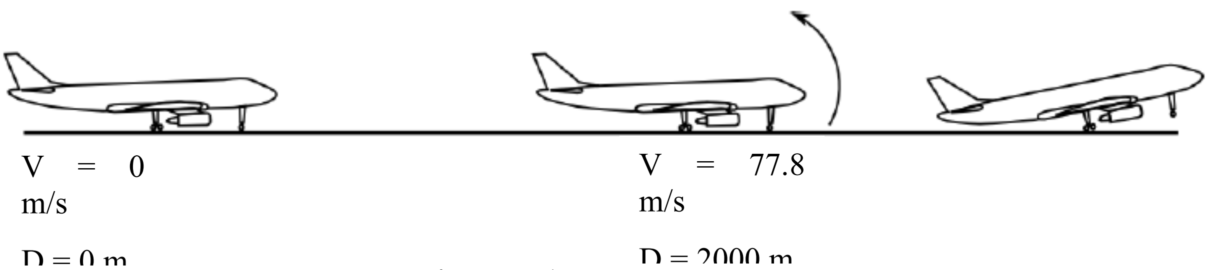


Figure 7-1: Take-off condition

We will now find the acceleration of the plane during the take-off by considering that the plane needs 50s to do 2000 m in this condition, we have:

$$\frac{dV}{dT} = \frac{V}{T} = \frac{77.8}{50} = 1.55\ m/s^2$$

The characteristic equation of the plane during take-off is given by:

$$\frac{dV}{dT} = \frac{F}{m} - \frac{\rho\, S\, V^2}{2\, m}\left(C_{x\ takeoff} - r\, C_{z\ takeoff}\right) - r\, g$$

Where r is the rolling resistance coefficient. It depends of the runway quality. For a paved and dry runway r=0.03. We will consider that there is no wind, we are at sea level and in a standard atmosphere (ρ=1.225).

$$C_{z\ takeoff} = \frac{2\, m\, g}{\rho\, S\, V^2} = \frac{2 * 560000 * 9.81}{1.225 * 845 * 77.8^2} = 1.75$$

$$C_{x\ takeoff} = 0.125 = Cz^2/(pi\ AR\ e)$$

e=1.01 and AR=7.72

Then we have: $F = \left(\frac{dV}{dT} + r * g + \frac{\rho * S * V^2}{2 * m}\left(C_{xtakeoff} - r * C_{z\ takeoff}\right)\right) * m$

$$F = 1.24 * 10^6\ N$$

F give the total force needed to take-off the plane, as we know A380 have 4 engines which means that with this configuration one engine need a minimum of $F = \frac{1.24 * 10^6}{4} = 310\ kN$

This means that we will choose 4 GeneralElectric GP7200 as engine for our A380. We know that these four engines consume 2.9 liters for 100km per PAX. A380 is capable to reach 15000km with 853 passengers. Then our typical consumption ratio is $C_s = \frac{2{,}9 * 853}{100} = 24.7\ L/km$ which is 12% less than Boeing 747.

8 Fuel Consumption

$HCV_{Kerosene} = 46200\ kJ/kg$ [14]

$\rho_{Kerosene} = 810\ kg/m^3$

$V_{max,fuel} = 320\ m^3$

$F_{drag,total} = 456\ kN$

Assuming the A380's engines (Engine Alliance GP7200) are ideal and cruising at 262.5 m/s for an entire 8-hour flight,

Power required to travel at 262.5 m/s and overcome drag force:

$$P = Fv = (456000) \times 262.5 = 119.7\ MW$$

Energy required for full flight:

$$E_{req} = Pt = (119.7\times10^{6})\times(8\times3600) = 3.446\times10^{9}\ k\text{J}$$

Volume of Kerosene burned to produce E_{req}:

$$V_{fuel,req} = \frac{E_{req}}{HCV_{kero}\rho_{kero}} = \frac{3.446\times10^{9}}{46200\times810} = 92.1\ m^{3} = 92{,}100\ litres$$

Total mass of burned fuel:

$$m_{fuel} = \rho_{fuel}\times V_{fuel} = 810\times92.1 = 74{,}601\ kg$$

Mass flow rate for all engines per second of flight at 262.5 m/s:

$$\dot{m} = \frac{m_{fuel}}{t_{flight}} = \frac{74601}{(8\times3600)} = 2.59\ kg/s$$

For each engine:

$$\dot{m} = \frac{2.59\ kg/s}{4} = 0.6475\ kg/s$$

Fuel consumption (kg/s) per kilo-newton of thrust to overcome skin friction for all engines:

$$TSFC = \frac{2.59}{456} = 5.68\times10^{-3}\ \frac{kg}{s}/kN$$

If the aircraft is at the maximum fuel capacity of 320,000 litres [15] and cruising at 262.5 m/s, the longest possible flight time is:

$$m_{fuel} = \rho_{fuel}\times V_{fuel} = 810\times320 = 259200\ kg$$

$$E_{max} = HCV_{kerosene}\times m_{fuel} = 46200\times259200 = 11.975\times10^{9}\ kJ$$

$$t = \frac{E}{P} = \frac{11.975\times10^{12}}{105.368\times10^{6}} = 113649.3\ s = 31.57\ hours$$

9 Conclusion

This aircraft was designed based on the specifications (such as wing span and chord length) from the Airbus A380. The aerofoil utilised for the design was the NACA 2408. The expected real lift (C_l) and drag coefficient (C_d) for this shape were found from the Javafoil software.

Using this aerofoil, the lift force required for take-off was found to be 6976kN. To achieve this, 4 Engine Alliance GP7277 engines providing 374kN of thrust were needed. The take-off speed over a distance to 2.6km was 111m/s.

During cruising conditions, the flow is turbulent across the fuselage and wings. Cruising velocity is estimated to be 262.5 m/s and generates a total drag force of 456kN.

The aircraft is allowed to approach the runway at a maximum velocity of 71 m/s and has a deceleration range of 0.7517 to 3.048 m/s^2 upon landing, depending on the runway conditions. These decelerations lead to stopping times of between 24 and 95 seconds.

The estimated fuel consumption for a total eight-hour flight is 92,100 litres, with a thrust-specific-fuel-consumption of 5.68×10^{-3} kg/s/kN.

10 Distribution of Work

Clémence Carton

•Identify the Joukowski aerofoil which matches that geometry most closely

•Calculate the lift coefficient for this equivalent Joukowski aerofoil

•Calculate a sensible solution of wing design and engine combination to accelerate the aircraft to its take-off speed within around 2000 m (give or take depending on your choice of aircraft). For that also calculate a typical fuel consumption rate.

Rhodri Davies

•Knowing that the drag almost doubles when the wheels are down, estimate the total drag coefficient/force for take-off/landing and cruising conditions. For the following, consider that extending the wing flaps both increases the wing area and changes the effective angle of attack of that wing.

•Estimate a drag coefficient for the fuselage and tail.

Matthew Dryden

•Draw the airfoil section with chord line and camber line.

•Extend the results from aerofoil sections to a finite wing.

•Find or calculate the expected real lift and drag coefficient for your aerofoil.

Jonathan Fairfoull

•Calculate the landing conditions and the breaking force required to slow down the plane to a sensible rolling speed.

•Using that same design as above but not making use of extended wings, calculate the power demand and fuel consumption rate for cruising.

•Finally, calculate the total fuel consumption for a typical flight.

Roshenac Mitchell

•Estimate the skin friction for the wings and the fuselage.

•From this and the measured or calculated drag coefficient for the section calculate the drag coefficient for the wings.

Kyle Sutherland

•Assuming ’flat plate’ conditions, calculate the laminar-turbulent transition of the boundary layer, the thickness of the boundary layer at the trailing edge of the wing, and at the tail of the fuselage.

References

[1] E. Rathakrishan, "Lift Distribution," in *Theoretical Aerodynamics*, Singapore City, Wiley, 2013, pp. 615 - 617.

[2] E. Rathakrishan, "Lift Coefficient," in *Theoretical Aerodynamics*, Singapore City, Wiley, 2013, p. 53.

[3] E. Rathakrishan, "Downward Wash," in *Theoretical Aerodynamics*, Singapore City, Wiley, 2013, pp. 611 - 617.

[4] Airbus, "DIMENSIONS & KEY DATA," 2015. [Online]. Available: http://www.airbus.com/aircraftfamilies/passengeraircraft/a380family/specifications/. [Accessed 18 11 2015].

[5] A. Wade, "Going Ballistic: Bullet Trajectories," *Undergraduate Journal of Mathematical Modeling: One + Two,* p. 4, 2011.

[6] M. Weisman, "The Airplane of the Next Twenty Years," 22 03 2001. [Online]. Available: http://www.dept.aoe.vt.edu/~mason/Mason_f/A380Weisman.pdf. [Accessed 18 11 2015].

[7] Airbus, A380 Aircraft Characteristics, Airport and Maintenance Planning, 2014, pp. 32 - 293.

[8] M. Hepperle, "JavaFoil - Analysis of Airfoils," 27 January 2007. [Online]. Available: http://www.mh-aerotools.de/airfoils/javafoil.htm. [Accessed 16 November 2015].

[9] Heathrow Airport Holdings Ltd., "Facts and Figures," [Online]. Available: http://www.heathrow.com/company/company-news-and-information/company-information/facts-and-figures. [Accessed 16 11 2015].

[10] The Port Authority of New York and New Jersey, *Public Notice of Passenger Facility Charge (PFC) Amendments for Newark Liberty Airport (EWR), LaGuardia Airport (LGA), John F. Kennedy International Airport (JFK), and Stewart International Airport (SWF),* New York: Port Authority, 2014.

[11] General Electric, "GE-Pratt & Whitney Engine Alliance Begins GP7000 Design," General Electric, 22 July 2002. [Online]. Available: http://www.geaviation.com/press/gepw/gepw_20020722l.html. [Accessed 16 November 2015].

[12] General Electric, "The GE90 Engine," General Electric, [Online]. Available: http://www.geaviation.com/commercial/engines/ge90/. [Accessed 16 November 2015].

[13] Airliners.net, 15 November 2015. [Online]. Available: http://www.airliners.net/aviation-forums/tech_ops/read.main/158906/.

[14] E. Toolbox, 18 November 2015. [Online]. Available: http://www.engineeringtoolbox.com/fuels-higher-calorific-values-d_169.html.

[15] Quora, 20 November 2015. [Online]. Available: https://www.quora.com/What-is-the-fuel-capacity-of-an-Airbus-A380.

[16] "Faa Safety," 20 November 2015. [Online]. Available: https://www.faasafety.gov/gslac/ALC/course_context.aspx?cID=34&sID=169&preview=true.

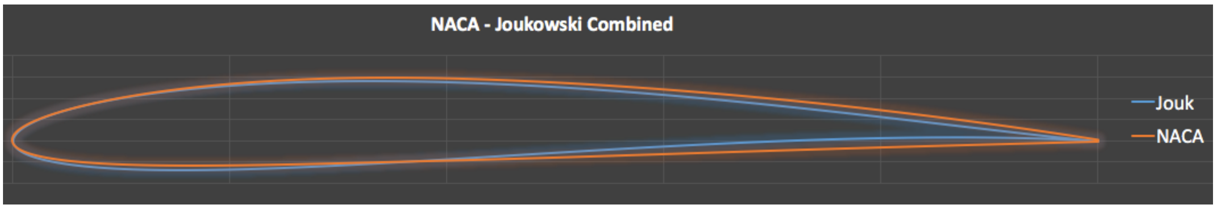
Appendices

Appendix A – Finite Wing (b/2)

distance from centre (m)	Distribution (Fl)	Induced Drag (fdi)	Lift Coef	Induced drag (Cdi)	
40	0	0	0.0000	0.0000	
39	4986	-34	0.0439	0.0001	
38	7007	-48	0.0616	0.0002	
37	8526	-58	0.0750	0.0003	
36	9781	-66	0.0860	0.0004	
35	10863	-74	0.0955	0.0004	
34	11820	-80	0.1040	0.0005	
33	12681	-86	0.1115	0.0006	
32	13463	-91	0.1184	0.0007	
31	14180	-96	0.1247	0.0008	
30	14842	-101	0.1305	0.0008	
29	15455	-105	0.1359	0.0009	
28	16025	-109	0.1409	0.0010	
27	16556	-112	0.1456	0.0010	
26	17052	-116	0.1500	0.0011	
25	17516	-119	0.1541	0.0012	
24	17951	-122	0.1579	0.0012	
23	18358	-125	0.1615	0.0013	
22	18740	-127	0.1648	0.0013	
21	19098	-130	0.1680	0.0014	
20	19433	-132	0.1709	0.0014	
19	19746	-134	0.1737	0.0015	
18	20039	-136	0.1762	0.0015	
17	20312	-138	0.1786	0.0016	
16	20566	-140	0.1809	0.0016	
15	20801	-141	0.1830	0.0016	
14	21020	-143	0.1849	0.0017	
13	21221	-144	0.1866	0.0017	
12	21405	-145	0.1883	0.0017	
11	21574	-146	0.1898	0.0018	
10	21726	-147	0.1911	0.0018	
9	21864	-148	0.1923	0.0018	
8	21986	-149	0.1934	0.0018	
7	22093	-150	0.1943	0.0018	
6	22185	-151	0.1951	0.0019	
5	22263	-151	0.1958	0.0019	
4	22326	-151	0.1964	0.0019	**Fuselage**
3	22376	-152	0.1968	0.0019	
2	22411	-152	0.1971	0.0019	
1	22432	-152	0.1973	0.0019	

Appendix B – Aerofoil

Figure B.1: Combined Aerofoil cross-section



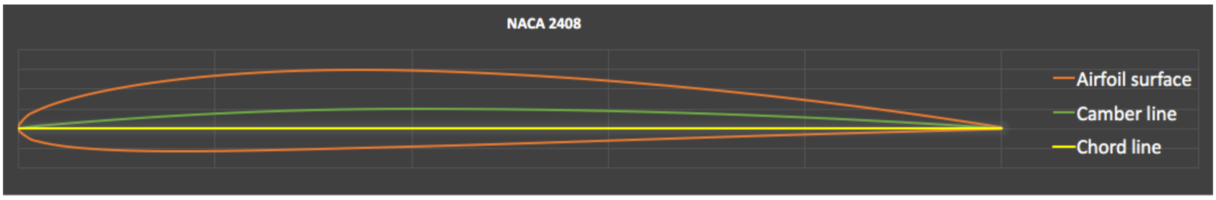


Figure B.3: NACA aerofoil

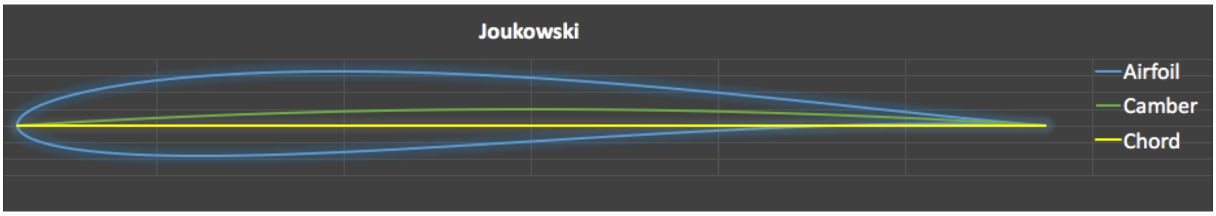


Figure B.2: Joukowski aerofoil

Appendix C – Matlab code for Joukowski Transform

```
clear all, clc
%Parameter -a is the angle of attack:
a=0; %in degrees
a=a*pi/180; %conversion to radians
%Parameter related to thickness of the airfoil:
t=0.08;
%Paramter related to camber of the airfoil:
c=0.02;
%Coordinates of circle of radius:
%R>1 in zp - plane
R=1+t;
theta = 0:pi/200:2*pi;
yp=R*sin(theta);
xp=R*cos(theta);

%plot of circle
figure(1)
plot((xp-t)*11/2,(yp+c)*11/2)
hold on
%Transformation circle from zp-plane to z-plane:
z=(xp-t)+i.*(yp+c);
%Joukowski transformation: z-plane to w-plane
rot=exp(i*a);
w=0.5.*rot.*(z+1./z);
% Plot of airfoil in w-plane
plot(real(w)*11,imag(w)*11,'r'), axis image

%NACA profile importation
ximport=[1.00000000
```

0.99981067	0.31509785	0.12725620
0.99907527	0.30044034	0.13779684
0.99785039	0.28598228	0.14868535
0.99613717	0.27173865	0.15991127
0.99393724	0.25772420	0.17146386
0.99125268	0.24395344	0.18333210
0.98808603	0.23044064	0.19550474
0.98444028	0.21719977	0.20797024
0.98031889	0.20424450	0.22071685
0.97572577	0.19158821	0.23373256
0.97066528	0.17924391	0.24700514
0.96514224	0.16722427	0.26052213
0.95916188	0.15554162	0.27427085
0.95272991	0.14420786	0.28823843
0.94585246	0.13323453	0.30241177
0.93853610	0.12263273	0.31677760
0.93078782	0.11241315	0.33132245
0.92261503	0.10258604	0.34603267
0.91402557	0.09316120	0.36089445
0.90502767	0.08414797	0.37589381
0.89563000	0.07555524	0.39101661
0.88584158	0.06739142	0.40628233
0.87567185	0.05966442	0.42169056
0.86513062	0.05238169	0.43717732
0.85422807	0.04555020	0.45272757
0.84297475	0.03917638	0.46832621
0.83138157	0.03326622	0.48395806
0.81945976	0.02782518	0.49960787
0.80722090	0.02285823	0.51526040
0.79467689	0.01836983	0.53090035
0.78183995	0.01436396	0.54651242
0.76872259	0.01084409	0.56208134
0.75533760	0.00781318	0.57759184
0.74169807	0.00527372	0.59302869
0.72781734	0.00322769	0.60837674
0.71370900	0.00167655	0.62362086
0.69938689	0.00062131	0.63874606
0.68486507	0.00006246	0.65373741
0.67015781	0.00000000	0.66858011
0.65527958	0.00043098	0.68325948
0.64024504	0.00135196	0.69776100
0.62506902	0.00276148	0.71207029
0.60976651	0.00465761	0.72617316
0.59435262	0.00703793	0.74005560
0.57884263	0.00989957	0.75370382
0.56325189	0.01323915	0.76710421
0.54759589	0.01705288	0.78024343
0.53189017	0.02133649	0.79310836
0.51615036	0.02608526	0.80568615
0.50039213	0.03129405	0.81796423
0.48463118	0.03695729	0.82993030
0.46888327	0.04306899	0.84157235
0.45316411	0.04962275	0.85287871
0.43748945	0.05661178	0.86383801
0.42187498	0.06402890	0.87443922
0.40633635	0.07186656	0.88467167
0.39084015	0.08011683	0.89452502
0.37541630	0.08877145	0.90398932
0.36011444	0.09782181	0.91305501
0.34495033	0.10725895	0.92171290
0.32993963	0.11707361	0.92995421

```
0.93777058
0.94515406
0.95209714
0.95859275
0.96463425
0.97021548
0.97533074
0.97997480
0.98414288
0.98783074
0.99103457
0.99375110
0.99597753
0.99771157
0.99895145
0.99969589
1.00000000];
yimport=[0.00000000
0.00003963
0.00015853
0.00035630
0.00063233
0.00098579
0.00141559
0.00192045
0.00249883
0.00314901
0.00386906
0.00465685
0.00551006
0.00642623
0.00740269
0.00843666
0.00952522
0.01066531
0.01185377
0.01308733
0.01436264
0.01567629
0.01702476
0.01840453
0.01981199
0.02124353
0.02269548
0.02416419
0.02564598
0.02713716
0.02863405
0.03013299
0.03163031
0.03312238
0.03460559
0.03607633
0.03753104
0.03896620
0.04037830
0.04176390
0.04311957
0.04444194
0.04572769
0.04697354
0.04817630
0.04933280
0.05043996
0.05149478
0.05249433
0.05343577
0.05431637
0.05513348
0.05588460
0.05656731
0.05717938
0.05771868
0.05818325
0.05856557
0.05883964
0.05900198
0.05905228
0.05899052
0.05881706
0.05853257
0.05813808
0.05763493
0.05702483
0.05630981
0.05549224
0.05457480
0.05356048
0.05245259
0.05125471
0.04997069
0.04860464
0.04716090
0.04564402
0.04405873
0.04240992
0.04070262
0.03894196
0.03713313
0.03528140
0.03339199
0.03147015
0.02952106
0.02754979
0.02556131
0.02356043
0.02155179
0.01953979
0.01752861
0.01552213
0.01352396
0.01153739
0.00956535
0.00761042
0.00567481
0.00376032
0.00186839
0.00000000
-0.00181875
-0.00356263
-0.00523133
-0.00682458
-0.00834218
-0.00978398
-0.01114988
-0.01243988
-0.01365402
-0.01479246
-0.01585540
-0.01684320
-0.01775628
-0.01859520
-0.01936064
-0.02005342
-0.02067449
-0.02122498
-0.02170614
-0.02211941
-0.02246639
-0.02274885
-0.02296874
-0.02312818
-0.02322948
-0.02327510
-0.02326768
-0.02321002
-0.02310508
-0.02295595
-0.02276587
-0.02253821
-0.02227642
-0.02198406
-0.02166475
-0.02132220
-0.02096013
-0.02058227
-0.02019238
-0.01979416
-0.01939130
-0.01898741
-0.01858599
-0.01818772
-0.01777155
-0.01733450
-0.01687915
-0.01640802
-0.01592356
-0.01542814
-0.01492405
-0.01441346
-0.01389846
-0.01338099
-0.01286289
-0.01234586
-0.01183148
-0.01132120
-0.01081632
-0.01031804
-0.00982740
-0.00934536
-0.00887272
-0.00841018
-0.00795836
```

```
-0.00751774
-0.00708875
-0.00667170
-0.00626686
-0.00587441
-0.00549450
-0.00512720
-0.00477258
-0.00443065
-0.00410140
-0.00378482
-0.00348087
-0.00318952
-0.00291074
-0.00264450
-0.00239077
-0.00214955
-0.00192083
-0.00170464
-0.00150100
-0.00130996
-0.00113157
-0.00096590
-0.00081303
-0.00067303
-0.00054601
-0.00043204
-0.00033122
-0.00024364
-0.00016938
-0.00010851
-0.00006110
-0.00002719
-0.00000684
0.00000000];

xn=2*ximport;
yn=2*yimport;
plot((xn-1)*11,yn*11,'g')
axis([-11.5 11.5 -10 10])
legend('joukowski-circle', 'jourkowski-airfoil', 'NACA2408')
title('Comparison between joukowski and NACA arfoil');
```

Appendix D – Total drag coefficient

Wing span	Lift Coef	Induced drag (Cdi)	Total Drag Coeff (Ct)	
40	0.0000	0.0000	0.00000	
39	0.1168	0.0007	0.01105	
38	0.1642	0.0013	0.01170	
37	0.1998	0.0019	0.01233	
36	0.2292	0.0026	0.01294	
35	0.2545	0.0032	0.01354	
34	0.2770	0.0037	0.01412	
33	0.2971	0.0043	0.01468	
32	0.3155	0.0048	0.01523	
31	0.3323	0.0054	0.01576	
30	0.3478	0.0059	0.01627	
29	0.3621	0.0064	0.01677	
28	0.3755	0.0069	0.01725	
27	0.3879	0.0073	0.01771	
26	0.3995	0.0078	0.01815	
25	0.4104	0.0082	0.01858	
24	0.4206	0.0086	0.01899	
23	0.4302	0.0090	0.01939	
22	0.4391	0.0094	0.01977	
21	0.4475	0.0097	0.02013	
20	0.4553	0.0101	0.02047	
19	0.4627	0.0104	0.02080	
18	0.4695	0.0107	0.02111	
17	0.4759	0.0110	0.02140	
16	0.4819	0.0113	0.02168	
15	0.4874	0.0116	0.02194	
14	0.4925	0.0118	0.02219	
13	0.4972	0.0120	0.02241	
12	0.5015	0.0122	0.02262	
11	0.5055	0.0124	0.02282	
10	0.5091	0.0126	0.02299	
9	0.5123	0.0128	0.02315	
8	0.5151	0.0129	0.02330	
7	0.5177	0.0130	0.02342	
6	0.5198	0.0131	0.02353	
5	0.5216	0.0132	0.02362	
4	0.5231	0.0133	0.02370	Fuselage
3	0.5243	0.0134	0.02376	
2	0.5251	0.0134	0.02380	
1	0.5256	0.0134	0.02382	
0	0.5258	0.0134	0.02383	

Appendix E

Appendix E.1 – Main Take-off Program

```
%This program is designed to model an aircraft take-off from stationary to
%the moment it leaves the ground modelling the speed and total drag
%Author: Rhodri Davies
%Date Created: 05/11/2015
%Updated by:   Rhodri Davies
%Updated on:   18/11/2015
%%
%heathrow- height: 25m;   runway: 3902m;
%newark  - height: 5.49m; runway: 3353m;
%
%%
%Parameters used for the aircraft
%Mass of aircraft (total at take off), Length & diameter of fuselage
%wingspan & chord length of wings
%rear wingspan & chord length, height & chord length of top tailwing
%force of thrust for the engines, used for linear speed(increases with time
%%
%Parameters of surrounding air
%density and kinematic viscosity at ground level
%% Reynolds number for each section
%Re for fuselage length determines the regime of flow used for Cd
%% parameters from JavaFoil (angle of attack = 0)
%coefficient of drag: wings, rear wings (all 3)
%coefficient of lift: wings provide all lift at this stage
%%
%coefficient of drag of fuselage broken into two sections
%cockpit(front facing) Cd of bullet - area pi r^2
%fuselage length(perpendicular to flow) Cd calculated for flat plate of
%area = pi * D * L
%%
%--------------------------------------------edit
%laminar and transitional occur before t=1s and is disregarded
%if Re_ < 360000      %laminar flow
%    Cd_ = 1.382   * Re_^(-0.5);
%else
%    if Re_fus > 500000 %turbulent flow
%        Cd_ = 0.0711 * Re_^(-0.2);
%    else                  %transitional flow
%        Cd_ = 0.0711/Re_*(Re_ - 360000 + (38.9*360000^(5/8)))^(4/5);
%    end
%end
%%
clear all, clc
m = 577000;
g = 9.807;
%fuselage length, diameter, areas
l_fus = 72.72;
d_fus = 7.14;
a_fus = pi * d_fus * l_fus;
a_front = pi * (d_fus/2)^2;%frontal area
%wing length, chord, area
l_wing = 79.75;
c_wing = 11;
a_wing = l_wing * c_wing;
ar_wing = l_wing / c_wing;
Cl_wing = 0.248; %induced drag of tips (for accelerating to speed)
```

```
Cl_wing_flap = 1.56; %1.56 for flap deflection of 20 degrees (javafoil)
E = 0.9; %airbus wing efficiency factor is 0.9 recommend c wing
%rear wings length, chord, area
l_rear = 30.37;
c_rear = 7.5;
a_rear = l_rear * c_rear;
%vertical rear wing height, chord, area
l_vert = 14.59;
c_vert = 9.5;
a_vert = l_vert * c_vert;
%%
%balancing front and rear wings with Cg
D_Cg = 22.23;
D_Fw = 29.94;
D_Rw = 63.065;
d_1 = D_Fw - D_Cg;
d_2 = D_Rw - D_Cg;
C = 1/(1-(d_1/d_2));
%%
%air properties at ground level atmpospheric pressure
rho = 1.225;
mu = 1.73*10^-5;
%%
s_to = 3000; %km runway
v_to = ((2*C*m*g)/(Cl_wing_flap*rho*a_wing))^0.5; %take off velocity
t_to = s_to/v_to;
f_to = (v_to^2) * m / ( 2 * s_to );     %constant force required to take
off
p_to = v_to*f_to;
%%
%initial conditions
s  = 0;
ds = 0;
t  = 0;
u0 = 0;
a  = 0;
%%
F_take_off = C*m*g; %lift force required for take off

%%
%delete comment for desired result
F_thrust = 311000*4; %4 x GP7200 rated take-off power
%F_thrust = 374000*4; %4 x GP7277 rated take-off power
%F_thrust = 420000*4; %4 x GE90 used in boeing 777 highest rated engines.
%%
F_lift = 0;
F_tot  = 0;
speed  = 0;
time   = 0;
acceleration = 0;
force  = 0;
distance = 0;
energy = 0;
power = 0;
e=0;
while F_lift < F_take_off
%Reynolds number for each component
x_re = rho * u0 / mu; %constant for all Re
Re_fus  = x_re * l_fus;
Re_wing = x_re * c_wing;
Re_rear = x_re * c_rear;
```

```
Re_vert = x_re * c_vert;
%Coefficient of drag for each component
if Re_fus == 0
    Cd_fus_l = 0;
else
    Cd_fus_l = 0.0711*(Re_fus)^(-0.2);
end
Cd_fus_f = 0.295;
Cd_wing  = wing_drag(Re_wing) + (Cl_wing^2)/(pi * ar_wing * E);
Cd_rear = rear_drag(Re_rear);
Cd_vert = vert_drag(Re_vert);
%drag force for each component
P_dy = 0.5 * rho * u0^2; %dynamic pressure
Fd_fus = P_dy * (Cd_fus_l * a_fus + Cd_fus_f * a_front); %length and front
Fd_wing  = P_dy * Cd_wing  * 2 * a_wing; %twice area to
Fd_rear  = P_dy * Cd_rear  * 2 * a_rear; %account for top
Fd_vert  = P_dy * Cd_vert  * 2 * a_vert; %and bottom surfaces
%%
F_drag   = (Fd_fus + Fd_wing + Fd_rear + Fd_vert)*1.2;%1.2 for landing gear
contribution
F_lift   = Cl_wing_flap * 0.5 * rho * u0^2 * a_wing * 0.68; %0.75 from
finite wing analysis
%%
ds = 0.1*u0;
s  = s + ds; %distance = speed * time + current position
u0 = u0 + 0.1*a; %v = u + at where u is initial velocity for each loop
a  = (F_thrust-F_drag) / m;    %horizontal acceleration m / s sq
F_tot = F_tot + F_drag;
%energy = energy + F_thrust * u0 * t;
energy = [energy ; ];
power = [power ; energy/t];
acceleration = [acceleration ; a];
speed = [speed ; u0];
time = [time ; t];
distance = [distance ; s];
t=t+0.1;
end
%%

%delete comment block to display desired result
%plot(time , speed)
%plot(time , acceleration)
%plot(time , distance)
```

Appendix E.2 – Auxiliary Function: Front Wing Drag

```
%%
%This script is used to find the coefficient of drag at different reynolds
%numbers (and therefore speeds) of the airbus a380 with a naca 2408 airfoil
%on the wing section

function Cd_wing = wing_drag(Re_wing)
switch Re_wing
    case Re_wing <2000000
        Cd_wing = 0.00471;
    case Re_wing >2000000 && Re_wing <3000000
        Cd_wing = 0.00497;
    case Re_wing >3000000   && Re_wing <4000000
        Cd_wing = 0.00509;
    case Re_wing >4000000 && Re_wing <5000000
        Cd_wing = 0.0051;
    case Re_wing >5000000 && Re_wing <6000000
        Cd_wing = 0.00553;
    case Re_wing >6000000 && Re_wing <7000000
        Cd_wing = 0.00554;
    case Re_wing >7000000 && Re_wing <8000000
        Cd_wing = 0.00551;
    case Re_wing >8000000 && Re_wing <9000000
        Cd_wing = 0.00553;
    case Re_wing >9000000 && Re_wing <10000000
        Cd_wing = 0.00554;
    case Re_wing >10000000 && Re_wing <11000000
        Cd_wing = 0.00552;
    case Re_wing >11000000 && Re_wing <12000000
        Cd_wing = 0.00553;
    case Re_wing >12000000 && Re_wing <13000000
        Cd_wing = 0.00552;
    case Re_wing >13000000 && Re_wing <14000000
        Cd_wing = 0.00552;
    case Re_wing >14000000 && Re_wing <15000000
        Cd_wing = 0.00553;
    case Re_wing >15000000 && Re_wing <16000000
        Cd_wing = 0.0055;
    case Re_wing >16000000 && Re_wing <17000000
        Cd_wing = 0.00552;
    case Re_wing >17000000 && Re_wing <18000000
        Cd_wing = 0.00552;
    case Re_wing >18000000 && Re_wing <19000000
        Cd_wing = 0.00552;
    case Re_wing >19000000 && Re_wing <20000000
        Cd_wing = 0.0055;
    case Re_wing >20000000 && Re_wing <25000000
        Cd_wing = 0.00548;
    case Re_wing >25000000 && Re_wing <30000000
        Cd_wing = 0.00547;
    case Re_wing >30000000 && Re_wing <35000000
        Cd_wing = 0.00542;
    case Re_wing >35000000 && Re_wing <40000000
        Cd_wing = 0.00541;
    case Re_wing >40000000 && Re_wing <45000000
        Cd_wing = 0.00536;
    case Re_wing >45000000 && Re_wing <50000000
        Cd_wing = 0.00533;
    case Re_wing >50000000 && Re_wing <55000000
```

```
        Cd_wing = 0.00529;
    case Re_wing >55000000 && Re_wing <60000000
        Cd_wing = 0.00528;
    case Re_wing >60000000 && Re_wing <65000000
        Cd_wing = 0.00522;
    case Re_wing >65000000 && Re_wing <70000000
        Cd_wing = 0.00518;
    case Re_wing >70000000 && Re_wing <75000000
        Cd_wing = 0.00515;
    case Re_wing >75000000 && Re_wing <80000000
        Cd_wing = 0.00512;
    case Re_wing >80000000 && Re_wing <85000000
        Cd_wing = 0.0051;
    case Re_wing >85000000 && Re_wing <90000000
        Cd_wing = 0.00509;
    case Re_wing >90000000 && Re_wing <95000000
        Cd_wing = 0.00504;
    case Re_wing >95000000 && Re_wing <100000000
        Cd_wing = 0.005;
    case Re_wing >100000000 && Re_wing <110000000
        Cd_wing = 0.00498;
    case Re_wing >110000000 && Re_wing <120000000
        Cd_wing = 0.00494;
    case Re_wing >120000000 && Re_wing <130000000
        Cd_wing = 0.00492;
    case Re_wing >130000000 && Re_wing <140000000
        Cd_wing = 0.00488;
    case Re_wing >140000000 && Re_wing <150000000
        Cd_wing = 0.00484;
    case Re_wing >150000000 && Re_wing <160000000
        Cd_wing = 0.0048;
    case Re_wing >160000000 && Re_wing <170000000
        Cd_wing = 0.00476;
    case Re_wing >170000000 && Re_wing <180000000
        Cd_wing = 0.00472;
    case Re_wing >180000000 && Re_wing <190000000
        Cd_wing = 0.0047;
    case Re_wing >190000000 && Re_wing <200000000
        Cd_wing = 0.00466;
    otherwise
        Cd_wing = 0.00462;
end;
```

Appendix E.3 – Auxiliary Function: Rear Vertical Wing Drag

```
%%
%This script is used to find the coefficient of drag at different reynolds
%numbers (and therefore speeds) of the airbus a380 with a naca 0008 airfoil
%on the rear vertical wing section
function Cd_vert = vert_drag(Re_vert)
switch Re_vert
case Re_vert <  2000000
    Cd_vert =   0.00473 ;
case Re_vert >  2000000 && Re_vert <    3000000
    Cd_vert =   0.00486 ;
case Re_vert >  3000000 && Re_vert <    4000000
    Cd_vert =   0.00503 ;
case Re_vert >  4000000 && Re_vert <    5000000
    Cd_vert =   0.00516 ;
case Re_vert >  5000000 && Re_vert <    6000000
    Cd_vert =   0.00527 ;
case Re_vert >  6000000 && Re_vert <    7000000
    Cd_vert =   0.00533 ;
case Re_vert >  7000000 && Re_vert <    8000000
    Cd_vert =   0.00536 ;
case Re_vert >  8000000 && Re_vert <    9000000
    Cd_vert =   0.0054  ;
case Re_vert >  9000000 && Re_vert <    10000000
    Cd_vert =   0.00544 ;
case Re_vert >  10000000    && Re_vert <    11000000
    Cd_vert =   0.00545 ;
case Re_vert >  11000000    && Re_vert <    12000000
    Cd_vert =   0.00547 ;
case Re_vert >  12000000    && Re_vert <    13000000
    Cd_vert =   0.00547 ;
case Re_vert >  13000000    && Re_vert <    14000000
    Cd_vert =   0.00548 ;
case Re_vert >  14000000    && Re_vert <    15000000
    Cd_vert =   0.00547 ;
case Re_vert >  15000000    && Re_vert <    16000000
    Cd_vert =   0.00549 ;
case Re_vert >  16000000    && Re_vert <    17000000
    Cd_vert =   0.00547 ;
case Re_vert >  17000000    && Re_vert <    18000000
    Cd_vert =   0.00548 ;
case Re_vert >  18000000    && Re_vert <    19000000
    Cd_vert =   0.00548 ;
case Re_vert >  19000000    && Re_vert <    20000000
    Cd_vert =   0.00547 ;
case Re_vert >  20000000    && Re_vert <    25000000
    Cd_vert =   0.00546 ;
case Re_vert >  25000000    && Re_vert <    30000000
    Cd_vert =   0.00543 ;
case Re_vert >  30000000    && Re_vert <    35000000
    Cd_vert =   0.00539 ;
case Re_vert >  35000000    && Re_vert <    40000000
    Cd_vert =   0.00536 ;
case Re_vert >  40000000    && Re_vert <    45000000
    Cd_vert =   0.00531 ;
case Re_vert >  45000000    && Re_vert <    50000000
    Cd_vert =   0.00525 ;
case Re_vert >  50000000    && Re_vert <    55000000
    Cd_vert =   0.00522 ;
case Re_vert >  55000000    && Re_vert <    60000000
```

```
    Cd_vert =   0.0052  ;
case Re_vert >  60000000    && Re_vert <    65000000
    Cd_vert =   0.00516 ;
case Re_vert >  65000000    && Re_vert <    70000000
    Cd_vert =   0.00513 ;
case Re_vert >  70000000    && Re_vert <    75000000
    Cd_vert =   0.00509 ;
case Re_vert >  75000000    && Re_vert <    80000000
    Cd_vert =   0.00505 ;
case Re_vert >  80000000    && Re_vert <    85000000
    Cd_vert =   0.00503 ;
case Re_vert >  85000000    && Re_vert <    90000000
    Cd_vert =   0.00499 ;
case Re_vert >  90000000    && Re_vert <    95000000
    Cd_vert =   0.00499 ;
case Re_vert >  95000000    && Re_vert <    100000000
    Cd_vert =   0.00495 ;
case Re_vert >  100000000   && Re_vert <    110000000
    Cd_vert =   0.00492 ;
case Re_vert >  110000000   && Re_vert <    120000000
    Cd_vert =   0.00487 ;
case Re_vert >  120000000   && Re_vert <    130000000
    Cd_vert =   0.00484 ;
case Re_vert >  130000000   && Re_vert <    140000000
    Cd_vert =   0.00479 ;
case Re_vert >  140000000   && Re_vert <    150000000
    Cd_vert =   0.00474 ;
case Re_vert >  150000000   && Re_vert <    160000000
    Cd_vert =   0.00474 ;
case Re_vert >  160000000   && Re_vert <    170000000
    Cd_vert =   0.00468 ;
case Re_vert >  170000000   && Re_vert <    180000000
    Cd_vert =   0.00465 ;
case Re_vert >  180000000   && Re_vert <    190000000
    Cd_vert =   0.00462 ;
case Re_vert >  190000000   && Re_vert <    200000000
    Cd_vert =   0.00462 ;
    otherwise
    Cd_vert =   0.00458 ;

end
```

Appendix E.4 – Auxiliary Function: Rear Horizontal Wing Drag

```
%%
%This script is used to find the coefficient of drag at different reynolds
%numbers (and therefore speeds) of the airbus a380 with a naca 0008 airfoil
%on the rear wing section
function Cd_rear = rear_drag(Re_rear)
switch Re_rear
    case Re_rear <  2000000
        Cd_rear =   0.00473 ;
case Re_rear >  2000000 && Re_rear <    3000000
    Cd_rear =   0.00486 ;
case Re_rear >  3000000 && Re_rear <    4000000
    Cd_rear =   0.00503 ;
case Re_rear >  4000000 && Re_rear <    5000000
    Cd_rear =   0.00516 ;
case Re_rear >  5000000 && Re_rear <    6000000
    Cd_rear =   0.00527 ;
case Re_rear >  6000000 && Re_rear <    7000000
    Cd_rear =   0.00533 ;
case Re_rear >  7000000 && Re_rear <    8000000
    Cd_rear =   0.00536 ;
case Re_rear >  8000000 && Re_rear <    9000000
    Cd_rear =   0.0054  ;
case Re_rear >  9000000 && Re_rear <    10000000
    Cd_rear =   0.00544 ;
case Re_rear >  10000000    && Re_rear <    11000000
    Cd_rear =   0.00545 ;
case Re_rear >  11000000    && Re_rear <    12000000
    Cd_rear =   0.00547 ;
case Re_rear >  12000000    && Re_rear <    13000000
    Cd_rear =   0.00547 ;
case Re_rear >  13000000    && Re_rear <    14000000
    Cd_rear =   0.00548 ;
case Re_rear >  14000000    && Re_rear <    15000000
    Cd_rear =   0.00547 ;
case Re_rear >  15000000    && Re_rear <    16000000
    Cd_rear =   0.00549 ;
case Re_rear >  16000000    && Re_rear <    17000000
    Cd_rear =   0.00547 ;
case Re_rear >  17000000    && Re_rear <    18000000
    Cd_rear =   0.00548 ;
case Re_rear >  18000000    && Re_rear <    19000000
    Cd_rear =   0.00548 ;
case Re_rear >  19000000    && Re_rear <    20000000
    Cd_rear =   0.00547 ;
case Re_rear >  20000000    && Re_rear <    25000000
    Cd_rear =   0.00546 ;
case Re_rear >  25000000    && Re_rear <    30000000
    Cd_rear =   0.00543 ;
case Re_rear >  30000000    && Re_rear <    35000000
    Cd_rear =   0.00539 ;
case Re_rear >  35000000    && Re_rear <    40000000
    Cd_rear =   0.00536 ;
case Re_rear >  40000000    && Re_rear <    45000000
    Cd_rear =   0.00531 ;
case Re_rear >  45000000    && Re_rear <    50000000
    Cd_rear =   0.00525 ;
case Re_rear >  50000000    && Re_rear <    55000000
    Cd_rear =   0.00522 ;
case Re_rear >  55000000    && Re_rear <    60000000
```

```
    Cd_rear =   0.0052  ;
case Re_rear >  60000000    && Re_rear <    65000000
    Cd_rear =   0.00516 ;
case Re_rear >  65000000    && Re_rear <    70000000
    Cd_rear =   0.00513 ;
case Re_rear >  70000000    && Re_rear <    75000000
    Cd_rear =   0.00509 ;
case Re_rear >  75000000    && Re_rear <    80000000
    Cd_rear =   0.00505 ;
case Re_rear >  80000000    && Re_rear <    85000000
    Cd_rear =   0.00503 ;
case Re_rear >  85000000    && Re_rear <    90000000
    Cd_rear =   0.00499 ;
case Re_rear >  90000000    && Re_rear <    95000000
    Cd_rear =   0.00499 ;
case Re_rear >  95000000    && Re_rear <    100000000

    Cd_rear =   0.00495 ;
case Re_rear >  100000000   && Re_rear <    110000000
    Cd_rear =   0.00492 ;
case Re_rear >  110000000   && Re_rear <    120000000
    Cd_rear =   0.00487 ;
case Re_rear >  120000000   && Re_rear <    130000000
    Cd_rear =   0.00484 ;
case Re_rear >  130000000   && Re_rear <    140000000
    Cd_rear =   0.00479 ;
case Re_rear >  140000000   && Re_rear <    150000000
    Cd_rear =   0.00474 ;
case Re_rear >  150000000   && Re_rear <    160000000
    Cd_rear =   0.00474 ;
case Re_rear >  160000000   && Re_rear <    170000000
    Cd_rear =   0.00468 ;
case Re_rear >  170000000   && Re_rear <    180000000
    Cd_rear =   0.00465 ;
case Re_rear >  180000000   && Re_rear <    190000000
    Cd_rear =   0.00462 ;
case Re_rear >  190000000   && Re_rear <    200000000
    Cd_rear =   0.00462 ;
    otherwise
        Cd_rear =   0.00458 ;

end
```

Appendix F – Landing Matlab code

```
% Deceleration calculations.
clear all; close all;
m = 395000; % Max landing mass
amax = 3.048; % Max deceleration
smax = 3353; % Total length of Newark
v = 71; % Max landing velocity
u = 0;  % Final velocity

smin = (v^2 - u^2)/(2*amax); % Stopping distance for max deceleration

amin = (v^2 - u^2)/(2*smax); % Deceleration for max stopping distance

tmax = sqrt((2*smax - u)/amin); % Time of max deceleration
tmin = sqrt((2*smin - u)/amax); % Time of min deceleration

Fbrakemax = m*(amax); % Total force of max deceleration
Fbrakemin = m*(amin); % Total force of min deceleration
```

Appendix F.1 – Landing Excel Spreadsheet

Landing conditions spread sheet available upon request. Screenshots of some calculated parameters from 100 m/s to 60 m/s:

D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	Re				Cd					Fd					
V (m/s)	Fuselage	Wing	Tail	Tail Wings	Fuselage	Wing	Tail	Tail Wings	Cd total	Cylinder	Front	Wing	Tail	Tail Wings	Fd total
100	516606243	78144509	67488439	53280347	0.001286	0.0018762	0.0019321	0.0020256	0.0071199	12890.255	72582.213	10139.65	1645.6009	4337.2909	121914.01
99	511440180	77363064	66813555	52747543	0.0012886	0.00188	0.001936	0.0020297	0.0071342	12659.159	71137.827	9957.8665	1616.0986	4259.5321	119556.58
98	506274118	76581618	66138671	52214740	0.0012912	0.0018838	0.0019399	0.0020338	0.0071487	12429.923	69707.958	9777.5464	1586.8338	4182.3992	117221.59
97	501108055	75800173	65463786	51681936	0.0012938	0.0018877	0.0019439	0.002038	0.0071634	12202.551	68292.605	9598.6923	1557.8069	4105.8934	114909.06
96	495941993	75018728	64788902	51149133	0.0012965	0.0018916	0.0019479	0.0020422	0.0071783	11977.046	66891.768	9421.3073	1529.0185	4030.016	112618.99
95	490775931	74237283	64114017	50616329	0.0012993	0.0018956	0.001952	0.0020465	0.0071933	11753.413	65505.448	9245.3943	1500.4689	3954.7682	110351.39
94	485609868	73455838	63439133	50083526	0.001302	0.0018996	0.0019561	0.0020508	0.0072086	11531.655	64133.644	9070.9565	1472.1588	3880.1515	108106.28
93	480443806	72674393	62764249	49550723	0.0013048	0.0019037	0.0019603	0.0020552	0.007224	11311.777	62776.356	8897.997	1444.0885	3806.1671	105883.66
92	475277743	71892948	62089364	49017919	0.0013076	0.0019078	0.0019646	0.0020597	0.0072397	11093.781	61433.585	8726.519	1416.2587	3732.8165	103683.55
91	470111681	71111503	61414480	48485116	0.0013105	0.001912	0.0019689	0.0020642	0.0072555	10877.673	60105.331	8556.5256	1388.6699	3660.1009	101505.96
90	464945618	70330058	60739595	47952312	0.0013134	0.0019162	0.0019732	0.0020687	0.0072715	10663.457	58791.593	8388.0202	1361.3225	3588.0217	99350.897
89	459779556	69548613	60064711	47419509	0.0013163	0.0019205	0.0019776	0.0020734	0.0072878	10451.137	57492.371	8221.0059	1334.2172	3516.5805	97218.374
88	454613494	68767168	59389827	46886705	0.0013193	0.0019248	0.0019821	0.0020781	0.0073043	10240.716	56207.666	8055.4862	1307.3544	3445.7785	95108.401
87	449447431	67985723	58714942	46353902	0.0013223	0.0019292	0.0019866	0.0020828	0.007321	10032.2	54937.477	7891.4644	1280.7347	3375.6173	93020.992
86	444281369	67204277	58040058	45821098	0.0013254	0.0019337	0.0019912	0.0020876	0.007338	9825.592	53681.805	7728.944	1254.3586	3306.0983	90956.158
85	439115306	66422832	57365173	45288295	0.0013285	0.0019382	0.0019959	0.0020925	0.0073551	9620.8973	52440.649	7567.9284	1228.2268	3237.223	88913.91
84	433949244	65641387	56690289	44755491	0.0013316	0.0019428	0.0020006	0.0020975	0.0073726	9418.1202	51214.01	7408.4212	1202.3398	3168.9929	86894.261
83	428783182	64859942	56015405	44222688	0.0013348	0.0019475	0.0020054	0.0021025	0.0073903	9217.2651	50001.887	7250.4258	1176.6982	3101.4095	84897.223
82	423617119	64078497	55340520	43689884	0.0013381	0.0019522	0.0020103	0.0021076	0.0074082	9018.3367	48804.28	7093.9461	1151.3025	3034.4744	82922.808
81	418451057	63297052	54665636	43157081	0.0013413	0.001957	0.0020152	0.0021128	0.0074264	8821.3397	47621.19	6938.9855	1126.1534	2968.1892	80971.03
80	413284994	62515607	53990751	42624277	0.0013447	0.0019619	0.0020203	0.0021181	0.0074449	8626.2788	46452.617	6785.548	1101.2514	2902.5554	79041.9
79	408118932	61734162	53315867	42091474	0.0013481	0.0019668	0.0020253	0.0021234	0.0074636	8433.1588	45298.559	6633.6371	1076.5973	2837.5748	77135.433
78	402952869	60952717	52640983	41558671	0.0013515	0.0019718	0.0020305	0.0021288	0.0074827	8241.9846	44159.019	6483.2569	1052.1915	2773.2488	75251.641
77	397786807	60171272	51966098	41025867	0.001355	0.0019769	0.0020358	0.0021343	0.007502	8052.7612	43033.994	6334.4112	1028.0348	2709.5792	73390.537
76	392620745	59389827	51291214	40493064	0.0013585	0.0019821	0.0020411	0.0021399	0.0075216	7865.4935	41923.486	6187.104	1004.1278	2646.5677	71552.135
75	387454682	58608382	50616329	39960260	0.0013622	0.0019874	0.0020465	0.0021456	0.0075416	7680.1869	40827.495	6041.3392	980.47107	2584.2161	69736.45
74	382288620	57826936	49941445	39427457	0.0013658	0.0019927	0.002052	0.0021513	0.0075619	7496.8463	39746.02	5897.1211	957.06537	2522.526	67943.495
73	377122557	57045491	49266561	38894653	0.0013695	0.0019981	0.0020576	0.0021572	0.0075825	7315.4772	38679.062	5754.4537	933.91136	2461.4992	66173.284
72	371956495	56264046	48591676	38361850	0.0013733	0.0020037	0.0020633	0.0021632	0.0076034	7136.0849	37626.619	5613.3412	911.0097	2401.1376	64425.831
71	366790432	55482601	47916792	37829046	0.0013772	0.0020093	0.0020691	0.0021692	0.0076247	6958.6748	36588.694	5473.788	888.36111	2341.4429	62701.153
70	361624370	54701156	47241908	37296243	0.0013811	0.002015	0.0020749	0.0021754	0.0076464	6783.2525	35565.285	5335.7985	865.96628	2282.4171	60999.263
69	356458308	53919711	46567023	36763439	0.0013851	0.0020208	0.0020809	0.0021817	0.0076684	6609.8237	34556.392	5199.377	843.82594	2224.0621	59320.177
68	351292245	53138266	45892139	36230636	0.0013891	0.0020267	0.002087	0.002188	0.0076908	6438.3941	33562.015	5064.5282	821.94082	2166.3798	57663.91
67	346126183	52356821	45217254	35697832	0.0013932	0.0020327	0.0020932	0.0021945	0.0077137	6268.9695	32582.156	4931.2565	800.31168	2109.3721	56030.478
66	340960120	51575376	44542370	35165029	0.0013974	0.0020388	0.0020995	0.0022011	0.0077369	6101.5559	31616.812	4799.5667	778.93926	2053.041	54419.898
65	335794058	50793931	43867486	34632225	0.0014017	0.0020451	0.0021059	0.0022079	0.0077606	5936.1594	30665.985	4669.4636	757.82434	1997.3887	52832.185
64	330627995	50012486	43192601	34099422	0.0014061	0.0020514	0.0021125	0.0022147	0.0077847	5772.7861	29729.675	4540.9519	736.96771	1942.4171	51267.357
63	325461933	49231040	42517717	33566618	0.0014105	0.0020579	0.0021191	0.0022217	0.0078092	5611.4423	28807.881	4414.0367	716.37018	1888.1285	49725.43
62	320295871	48449595	41842832	33033815	0.001415	0.0020645	0.0021259	0.0022288	0.0078342	5452.1344	27900.603	4288.7229	696.03255	1834.5248	48206.421
61	315129808	47668150	41167948	32501012	0.0014196	0.0020712	0.0021328	0.0022361	0.0078598	5294.869	27007.842	4165.0158	675.95566	1781.6084	46710.348
60	309963746	46886705	40493064	31968208	0.0014243	0.0020781	0.0021399	0.0022435	0.0078858	5139.6526	26129.597	4042.9205	656.14037	1729.3815	45237.23

U	V	W	X	Y	Z	AA	AB	AC	AD	AE
Fl Wing										
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
-3799167	-3599211	-3237128	-2799386	-2296794	-1745563	-1156503	-540422	81063.304	707952.85	1340246.62
-3723563	-3527586	-3172709	-2743678	-2251087	-1710826	-1133489	-529667.6	79450.144	693864.59	1313575.71
-3648720	-3456682	-3108938	-2688530	-2205841	-1676439	-1110706	-519021.3	77853.197	679917.92	1287172.86
-3574636	-3386497	-3045814	-2633942	-2161053	-1642400	-1088154	-508483.1	76272.462	666112.84	1261038.05
-3501312	-3317033	-2983337	-2579914	-2116725	-1608711	-1065833	-498052.9	74707.941	652449.35	1235171.29
-3428748	-3248288	-2921508	-2526446	-2072856	-1575371	-1043744	-487730.9	73159.632	638927.45	1209572.58
-3356944	-3180263	-2860326	-2473538	-2029447	-1542380	-1021886	-477516.9	71627.535	625547.14	1184241.92
-3285899	-3112957	-2799792	-2421189	-1986497	-1509738	-1000260	-467411	70111.651	612308.42	1159179.3
-3215615	-3046372	-2739905	-2369400	-1944006	-1477445	-978864.3	-457413.2	68611.98	599211.29	1134384.74
-3146090	-2980506	-2680666	-2318172	-1901975	-1445501	-957700.2	-447523.5	67128.522	586255.76	1109858.23
-3077325	-2915361	-2622074	-2267503	-1860403	-1413906	-936767.5	-437741.8	65661.276	573441.81	1085599.76
-3009320	-2850935	-2564129	-2217394	-1819290	-1382661	-916066.1	-428068.3	64210.243	560769.45	1061609.35
-2942075	-2787229	-2506832	-2167845	-1778637	-1351764	-895596	-418502.8	62775.422	548238.69	1037886.98
-2875589	-2724243	-2450182	-2118855	-1738443	-1321217	-875357.2	-409045.4	61356.815	535849.51	1014432.67
-2809864	-2661976	-2394180	-2070426	-1698709	-1291018	-855349.7	-399696.1	59954.419	523601.93	991246.402
-2744898	-2600430	-2338825	-2022556	-1659433	-1261169	-835573.5	-390454.9	58568.237	511495.94	968328.184
-2680692	-2539603	-2284117	-1975247	-1620618	-1231669	-816028.6	-381321.8	57198.267	499531.53	945678.016
-2617246	-2479496	-2230057	-1928497	-1582261	-1202518	-796715	-372296.7	55844.51	487708.72	923295.898
-2554560	-2420109	-2176645	-1882307	-1544364	-1173717	-777632.7	-363379.8	54506.965	476027.5	901181.829
-2492633	-2361442	-2123880	-1836677	-1506926	-1145264	-758781.7	-354570.9	53185.634	464487.87	879335.809
-2431467	-2303495	-2071762	-1791607	-1469948	-1117160	-740162	-345870.1	51880.514	453089.83	857757.838
-2371060	-2246267	-2020292	-1747097	-1433429	-1089406	-721773.6	-337277.4	50591.608	441833.38	836447.917
-2311413	-2189760	-1969469	-1703146	-1397369	-1062001	-703616.5	-328792.8	49318.914	430718.52	815406.045
-2252526	-2133972	-1919293	-1659756	-1361769	-1034944	-685690.7	-320416.2	48062.433	419745.25	794632.222
-2194399	-2078904	-1869765	-1616925	-1326628	-1008237	-667996.2	-312147.8	46822.164	408913.57	774126.449
-2137031	-2024556	-1820884	-1574655	-1291946	-981879.3	-650533	-303987.4	45598.108	398223.48	753888.725
-2080424	-1970928	-1772651	-1532944	-1257724	-955870.4	-633301.1	-295935.1	44390.265	387674.98	733919.05
-2024576	-1918019	-1725065	-1491793	-1223961	-930210.6	-616300.5	-287990.9	43198.635	377268.08	714217.425
-1969488	-1865831	-1678127	-1451202	-1190658	-904899.9	-599531.2	-280154.8	42023.217	367002.76	694783.849
-1915160	-1814362	-1631836	-1411171	-1157814	-879938.4	-582993.2	-272426.7	40864.011	356879.03	675618.322
-1861592	-1763613	-1586193	-1371699	-1125429	-855325.9	-566686.5	-264806.8	39721.019	346896.9	656720.845
-1808783	-1713584	-1541197	-1332788	-1093503	-831062.6	-550611.1	-257294.9	38594.239	337056.35	638091.417
-1756735	-1664275	-1496848	-1294436	-1062037	-807148.4	-534767	-249891.1	37483.672	327357.4	619730.038
-1705446	-1615686	-1453147	-1256644	-1031031	-783583.3	-519154.3	-242595.4	36389.317	317800.04	601636.709
-1654917	-1567816	-1410093	-1219413	-1000483	-760367.3	-503772.8	-235407.8	35311.175	308384.26	583811.429
-1605148	-1520667	-1367687	-1182741	-970395.3	-737500.4	-488622.6	-228328.3	34249.246	299110.08	566254.198
-1556139	-1474237	-1325928	-1146629	-940766.7	-714982.7	-473703.7	-221356.9	33203.529	289977.49	548965.016
-1507889	-1428527	-1284816	-1111076	-911597.4	-692814	-459016.1	-214493.5	32174.025	280986.49	531943.884
-1460400	-1383537	-1244352	-1076084	-882887.5	-670994.5	-444559.8	-207738.2	31160.734	272137.08	515190.801
-1413670	-1339266	-1204535	-1041652	-854636.9	-649524	-430334.8	-201091	30163.655	263429.26	498705.768
-1367700	-1295716	-1165366	-1007779	-826845.7	-628402.7	-416341.1	-194551.9	29182.789	254863.03	482488.784

AM	AN	AO	AP
			a = -3.048
Fd Induced	Fd total 20˚ (Fd)	Fbrakes	Fbrake
1021063.707	1501494.874	-301494.874	-18843.43
1000744.54	1471659.788	-271659.788	-16978.74
980629.5846	1442123.74	-242123.74	-15132.73
960718.8423	1412886.736	-212886.736	-13305.42
941012.3128	1383948.782	-183948.782	-11496.8
921509.9959	1355309.886	-155309.886	-9706.868
902211.8919	1326970.054	-126970.054	-7935.628
883118.0005	1298929.294	-98929.2935	-6183.081
864228.322	1271187.611	-71187.6113	-4449.226
845542.8561	1243745.015	-43745.0147	-2734.063
827061.603	1216601.511	-16601.5109	-1037.594
808784.5626	1189757.107	10242.89283	640.1808
790711.735	1163211.811	36788.1889	2299.2618
772843.1201	1136965.63	63034.36981	3939.6481
755178.718	1111018.572	88981.42789	5561.3392
737718.5286	1085370.645	114629.3554	7164.3347
720462.552	1060021.856	139978.1444	8748.634
703410.788	1034972.213	165027.7871	10314.237
686563.2369	1010221.725	189778.2751	11861.142
669919.8984	985770.3995	214229.6005	13389.35
653480.7727	961618.2453	238381.7547	14898.86
637245.8598	937765.2706	262234.7294	16389.671
621215.1596	914211.4841	285788.5159	17861.782
605388.6721	890956.8944	309043.1056	19315.194
589766.3974	868001.5105	331998.4895	20749.906
574348.3354	845345.3412	354654.6588	22165.916
559134.4862	822988.3959	377011.6041	23563.225
544124.8497	800930.6836	399069.3164	24941.832
529319.4259	779172.214	420827.786	26301.737
514718.2149	757712.9965	442287.0035	27642.938
500321.2166	736553.041	463446.959	28965.435
486128.4311	715692.3573	484307.6427	30269.228
472139.8583	695130.9556	504869.0444	31554.315
458355.4983	674868.8461	525131.1539	32820.697
444775.351	654906.0392	545093.9608	34068.373
431399.4164	635242.5457	564757.4543	35297.341
418227.6946	615878.3764	584121.6236	36507.601
405260.1855	596813.5422	603186.4578	37699.154
392496.8891	578048.0545	621951.9455	38871.997